

THE
Practical Gager;

OR, THE
YOUNG GAGER'S ASSISTANT.

CONTAINING

Those Things which are actually practised,
and which are also absolutely necessary to be
known and understood by every Person that
is employed as a Gager, or Officer in the
Revenue of Excise.

To which are added

All the necessary Tables for gaging and fixing the
Utensils of Victuallers, Common Brewers, and also
for moneying the several Sorts of Goods, or for
finding the Amounts of the Charges.

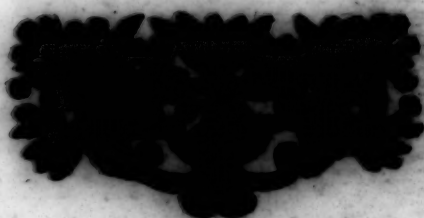
*The Whole is a Method entirely new; intended chiefly
for the Help of Pupils, and such young Officers as
have not been long employed in the Office of Excise.*

With an APPENDIX.

Bonum quod communis, est melius.

By WILLIAM SYMONS,

Officer of EXCISE.



LONDON:

Printed for J. Nourse, opposite Kew's
Street in the Strand.
MDCCLV.

Practical Gager;

YOUNG GAGERS ASSISTANT.

That which is the most useful and necessary in the art of Gaging, is the knowledge of the several sorts of Gages, and the manner of using them, which is the subject of this little Treatise.

To which is added

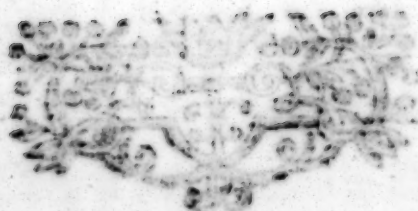
All the necessary Tables for finding the Content of any Solid, and the several sorts of Gages, and the manner of using them.

By WILLIAM SYMONS, Surveyor-General of the Ordnance, and Master of the Mint.

IN TWO VOLUMES.

Price 10s. 6d. per Volume.

By WILLIAM SYMONS,



LONDON:

Printed for J. Molesworth, opposite Aldgate, in the Strand.



THE P R E F A C E.

To the Candid R E A D E R.

IN the following Pages I have collected together all the most useful and necessary Directions that are practised by the Officers of Excise in the whole Art of Gaging, and given the most concise Rules for performing them. Throughout the Whole I have endeavoured to free the Art from all those Obscurities, and Mixtures of other Things, which are frequently met with in Books of Gaging, are foreign to the Purpose, and of no Service to the practical Gager, but to puzzle and perplex rather than to inform him.

I have proceeded gradually from Step to Step in every respect, reduced the Theory of the Art to the present Method of Practice, and have laid down the Whole in the plainest and easiest Manner, by giving familiar Rules, and Examples wrought both by Pen and Rule; so that any one but of an ordinary Capacity may readily comprehend them, and soon become Master of the whole Art.

The Methods of performing the Business of Gaging are not only here shown, but the Grounds and Reasons also why the Gager is to proceed in such and such a Manner; which will enable him to understand what he is about when he is upon his Duty. On those Heads where others have dwelt too long, I have been the shorter; and where they have been too brief, those Heads I have enlarged and explained.

In this Undertaking I have strictly adhered to those Methods which are now made use of by the Officers of Excise; and tho' many of the Examples are wrought by a different Process, yet the Solutions are exactly the same; so that the young Gager is left to his Choice in the Use of that Operation which he finds to be fittest for his Purpose.

I am sensible that there have already been a Multitude of Books written on this Subject by Persons of superior Abilities; many of whom have treated it in a very learned and judicious Manner, as far as it relates to the Theory of the Art. And others too have handled the practical Part of Gaging very well; that at first View one would be ready to imagine that there

could be nothing further needful written on this Head. But on a closer Review of the several Treatises extant on this Subject one does not find any of them so well adapted to the present Method of Practice as one might reasonably expect; for some of them are written in a Style beyond the Reach of a common Capacity to comprehend, unless he were acquainted with the algebraic Art; and others who have treated on the practical Parts of Gaging, have intirely neglected, or superficially passed over, several useful Things which are of absolute Necessity in Practice.

On this Account, and at the Instigation of some of my Friends in the Excise, I was prevailed upon to compile this Treatise, which, without any farther Apology, I here present the Reader with, relying upon his Candour in excusing the Inaccuracy of Style, or any other Errors I may have been guilty of in the Course of it.

I doubt not but many into whose Hands this Book may come, will be ready to object, "that they can see nothing in it but what every Officer of Excise must know;" and this I as readily acknowledge. But the Design of its Publication is not for the Information of such as know these Things already, but for the sake of young Beginners, and Candidates for the Excise. Besides it would be a Sign of Presumption in me to attempt to introduce new Methods of Gaging into Practice, when these already made use of, and which are enjoined by my Superiors, so fully answer the End to all Intents and Purposes.

My Intent was only to collect together, in as small a Compass as possible, every Thing that would be useful, necessary, or curious in the Art of Gaging; and to lay down Rules and Precedents to be applied upon every Emergency without the Help of any other Book.

How far I have answered this End is submitted to the practical Gager. And if what I have done shall prove of any Service to my Brother Officers, I shall think my Labour and Pains very well bestowed.

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ERRATA.

Pl. Line.

- 18 13 for so are, read so is the Content.
- 21 13 for Side, read Side.
- 39 22 after every third, add Figure.
- 43 24 } for { 10734676 } read { 10634676
- 25 } { 1376924 } { 1476924.
- 44 16 for 9.4247760, read 94.247760.
- ibid 19 for 94.24, read 94.24.
- 46 7 for 70.6858200, read 706.858200.
- ibid 10 for 70.68, read 706.8.
- 86 the Letter g omitted in the Figure of a Cooler.
- 88 3 read Art. ix. Chap. xiii.
- 89 9 for Areas, read Area.
- 99 Line last but 2, for 2.75, read 2.20.
- 101 9 after Area at 1 add Inch.
- 102 1 for needed, read exceeded.
- 111 6 for 0:1:2,258, read 0:1:0,258.
- 119 17 for 23.80, read 2.380.
- 120 last but 4, after Line dele 1.
- 124 last but 7, for 952.25, read 992.25.
- 125 24 after Diameter, add 1.
- 128 21 for 194, read 19.4.
- 131 3 for 48,2, read 48,1.
- 137 10 for each, read their.
- 156 29 for 3525, read 35,25.
- 158 16 for Sorts, read Sort.
- 171 15 for vi, read VI.

EXPLANATION of the CHARACTERS.

FOR the sake of shortening the Work I have made use of a few Algebraic Signs or Characters, which are explained as follows:

Signs or Characters.

+ more. { This Character is the Sign of Addition: As $6 + 5$ signifies that 6 and 5 are to be added together.

— less. { This Character is the Sign of Subtraction: As $9 - 4$ signifies that 4 is to be subtracted from 9.

x multiplied. { This Character is a Sign of Multiplication: As 7×3 signifies that 7 is to be multiplied by 3.

= equal. { This Character is a Sign of Equality: As $7 = 7$, or $6 + 5 = 11$, or $12 - 3 = 9$ signifies that 7 is equal to 7; 6 more 5 is equal to 11; and 12 less 3 is equal to 9.

: : so is } This Character is a Sign of Proportion, and is always put between the two middle Terms in the Rule of Three: Thus $4 : 8 :: 5 : 10$, which signifies that as 4 is to 8, so is 5 to 10.

X Strong. { This is the Character of Strong Beer.

VI Small. { This is the Character of Small Beer.

A. G.	} signifies	Ale Gallons.
W. G.		Wine Gallons.
M. B.		Malt Bushels.
G. P.		The Gagepoint.
S. G. P.		The Square Gagepoint.
C. G. P.		The Circular Gagepoint.

THE PRACTICAL GAGER.

CHAP. I.

Of DECIMAL FRACTIONS.

MY Design here is not to treat the Subject of Decimal Fractions at large, for that would require a Volume of itself larger than I intend this to be; besides, that has been already done by so many Hands, that I think it quite needless to add a great deal on that Head. Therefore I shall only give a few short and easy Rules, and subjoin some familiar Examples, to assist the Memory of those who have already learn'd it, which, if thoroughly understood, may be made applicable to all the various Cases of practical Gaging.

ART. I.

Of Notation of Decimals.

AS in whole Numbers the Value of every Figure from the Unit's Place to the left Hand increases in Value, in a decuple or tenfold Proportion; so in Decimals every Figure to the right Hand of Unity decreases in Value in a decuple or tenfold Proportion, as will appear by the following Table.

Whole Numbers. Decimals.

XCM.	XC.	XM.	X.	CM.	C.	X.	Units.	X.	Parts.	X.	Parts.	X.	Parts.	X.	Parts.	X.	Parts.
7	6	5	4	3	2	1	1	2	3	4	5	6	7	8	9	0	1

And whereas Cyphers at the left Hand or before whole Numbers signify nothing, as for Example, 00365 = 365, and 0456 = 456, so Cyphers at the right Hand or after Decimals signify nothing, as, .36500 is the same as .365, and .45600 = .456. But Cyphers at the left Hand of Decimals decrease their Value in a tenfold Proportion, as will appear by the following Table:

B

That,

2 Of Decimal Fractions.

Thus,

$$\left. \begin{array}{r} .5 \\ .05 \\ .005 \\ .0005 \end{array} \right\} \text{ is } \left\{ \begin{array}{l} \text{Tenth Parts} \\ \text{C Parts} \\ \text{M Parts} \\ \text{XM Parts} \end{array} \right\} \text{ of Unity.}$$

A R T. II.

Of Addition and Subtraction of Decimals.

THIS is performed in the same Manner as in common Arithmetic, only remember to place Primes under Primes, Seconds under Seconds, Thirds under Thirds, &c. as in the following Examples :

E X A M P L E S.

ADDITION.

$$\begin{array}{r} \text{To } 45,6053 \\ \text{Add } 34,0632 \\ \hline \end{array}$$

$$\text{Sum} = 79,6685$$

$$\begin{array}{r} \text{To } .006532 \\ \text{Add } .765039 \\ \hline \end{array}$$

$$\text{Sum} = .771571$$

SUBTRACTION.

$$\begin{array}{r} \text{From: } 23,05640 \\ \text{Subtract } 22,98764 \\ \hline \end{array}$$

$$\text{Diff.} = .06876$$

$$\begin{array}{r} \text{From } 6,540938 \\ \text{Subtract } 5,847603 \\ \hline \end{array}$$

$$\text{Diff.} = .693335$$

This being so easy that I shall add no more Examples.

A R T. III.

Multiplication of Decimals.

R U L E I.

Multiply both Factors together as in common Arithmetic; when done count how many Decimals there were in both Factors, and so many Figures prick off in the Product for Decimals.

II. If it should happen that there should not be so many Decimals in the Product, as there were in the two Factors, then add as many Cyphers to the left Hand of the Product as will make up the Deficiency. See the following Examples :

3

EXAMPLES.

Multiply	8,506	,007654
By	6,396	,003542
	51036	15308
	76554	30016
	17012	38270
	51036	22962
Product	53,553776	,000027110468

In the first Example there are three Decimals in each Factor; therefore I prick off six Figures in the Product for Decimals, and in the second Example there are six Decimals in each Factor, and their Sum is equal to twelve: But there being only eight Figures in the Product, therefore I add four Cyphers more to the left Hand of the Product, which makes the Number twelve, equal to the Number of both Factors, which is true. The like must be done in all Cases.

ART. IV.

Division of Decimals.

RULE I.

AS many more Decimals as there are in the Dividend than the Divisor, so many Figures prick off in the Quotient for Decimals.

II. But if there are not so many Decimals in the Dividend as there are in the Divisor, then add as many Cyphers to the right Hand of the Dividend as will make their Number equal, and in that Case the Quotient will be Integers, or whole Numbers. See the following Examples:

EXAMPLES.

[illegible]

Of Decimal Fractions.

In the first Example there are six Decimals in the Dividend, and none in the Divisor; consequently six Figures are to be pick'd off for Decimals in the Quotient. But there are only four Figures, viz. 2785 produced in the Quotient by Division; therefore I add two Cyphers to the left Hand of the Quotient, which will make it true. In the second Example there are four Decimals in the Divisor, and none in the Dividend; therefore I add four Cyphers to the Dividend, which makes their Number equal, and the Quotient comes out whole Numbers, viz. 359. The other two Figures are gain'd by adding Cyphers to the Remainder.

More EXAMPLES.

$$\begin{array}{r}
 25,6), 5065(,019 \\
 \underline{256} \\
 2505 \\
 \underline{2304} \\
 201
 \end{array}$$

$$\begin{array}{r}
 ,005) 725,000(145000 \\
 \underline{5} \\
 22 \\
 \underline{20} \\
 25 \\
 \underline{25}
 \end{array}$$

In the first of these Examples there are three Decimals more in the Dividend than in the Divisor; for which Reason there must be three Decimals in the Quotient: Therefore to the Quotient 19 I prefix a Cypher, which makes it true.

In the second Example the Divisor consists of three Places of Decimals; and as there are no Decimals in the Dividend, I add three Cyphers, which make their Numbers equal, and the Quotient comes out whole Numbers.

And thus, by observing these general Rules, the Value of the Quotient may be discovered in all Cases.

A R T. V.

Of Reduction of Decimals.

TO reduce a Vulgar Fraction to its equivalent Value in Decimals.

R U L E.

Divide the Numerator of the given Vulgar Fraction by its Denominator, and the Quotient will be the Decimal required.

E X A M P L E S.

What are the Decimals of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$?

Of Decimal Fractions.

5

OPERATION.

4)1,00,25	2)10,5	4)3,00,75
8	10	28
—	—	—
20	0	20
20		20
—		—
0		0

Thus I find the Decimal of $\frac{1}{4} = .25$, $\frac{1}{2} = .5$, and $\frac{3}{4} = .75$. In like Manner may any other Vulgar Fraction be reduced when the Numerator and Denominator consists of more than one Figure. As for Example, What is the Decimal of $\frac{17}{25}$?

OPERATION.

25)17,0(,68 = Decimal required.

150
—
200
200
—
0

Note, When there is a Remainder, add Cyphers to it, and continue the Division till you have gained six Places of Decimals in the Quotient, which will be sufficient for most Purposes.

A R T. VI.

TO reduce the different Denomination of Money, Weights, and Measures to their equivalent Decimal Expressions.

R U L E.

- I. Place the several Denominations orderly over one another, (the least uppermost) then
- II. Divide the lesser Denomination by the Number of Units of that Denomination contained in the next greater Denomination, and add the Quotient to the next greater Denomination.
- III. Divide this Number by the Number of Units of this Denomination contained in the next greater, and add the Quotient to the next greater Denomination, and so proceed to do till you arrive at the Decimal of the Integer, which will appear more plain by the following Examples :

E X A M P L E S.

Let it be required to reduce 10 s. 9 d. and 15 s. 3 d. 3 f. each to its equivalent Decimal of a Pound Sterling.

B 3

OPB

O P E R A T I O N.

Pence 12	9	Farthings 4	3
Shill. 20	10,75	Pence 12	6,75
	5375	Shill. 20	15,5625
			778125

By which I find the Decimal of 10 s. and 9 d. = .5375 l. and that of 15 s. 6 d. 3 f. = .778125. And in like Manner may any other Denomination (whether they be Weights or Measures) be reduced to their equivalent Decimal Expressions.

A R T. VII.

TO reduce or find the Value of any Decimal Fraction.

R U L E.

Multiply the given Decimal by the Number of Units of the next lesser Denomination contained in one of that Denomination which the Decimal respects for its Integer, and from that Product cut off so many Figures as there were in the given Decimal, and this Decimal multiply by the next lesser Denomination, and prick off the first Number of Decimals, and thus proceed to do till you have multiplied by all the Denominations contained in the Integer. Then the whole Numbers on the left Hand of the separating Point will be the Value of the Decimal required.

For Example, What is the Value of .5375 l. and .778125 l.?

O P E R A T I O N.

.5375	.778125
20	20
10,7500	15,562500
12	12
9,0000	6,750000
	4
	3,000000

By which I find the Value of .5375 = 10 s. 9 d. and that of .778125 = 15 s. 6 d. 3 f. which are the same as were required to be reduced in the last Article; and in this Manner may any other decimal Fraction be reduced. But the Value of the Decimal of a Pound Sterling may be discovered by Inspection only.

R U L E.

R U L E.

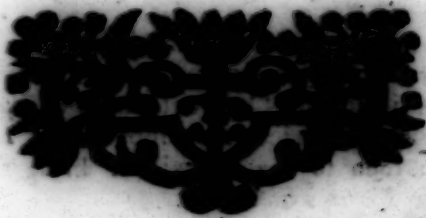
The Primes place double, and account so many Shillings; if the second Figure be five or more add one Shilling more to the Double of the first Figure. And if any Thing remain above five in the Place of Seconds, or if the Seconds are under five account them as so many Tens, to which said Tens add the Figure in the third Place, which will be so many Units. Their Sum account as one entire Number, which being made less by one in every twenty five will be Farthings, to be added to the Shillings before found, and this Sum will be equal to the Value of the given Decimal, which will be more intelligible by the following Examples :

E X A M P L E S.

What is the Value of $.9625 \text{ £}$.

The first Figure is 9, which doubled is 18; the second being 6 I take 5 from it, and add 1 to the 18, which makes 19s. The Remainder of the second Figure above 5 is 1, which is 10 Farthings, and the Figure in the third Place, being 2, I add to the 10, which makes 12 Farthings. The Figure in the fourth Place, being 5, or a Half Farthing, is abated according to the Rule; for if 25 abate 1, 12,5 will abate 5 Tenths. So I find the Value of the given Decimal to be equal to 19s. 3d.

In like Manner may the Value of the Decimal $.5375 \text{ £}$ be found equal to 10s. 9d. And that of the Decimal $.778125 \text{ £}$ equal to 15s. 6d. 3f. or the Value of any other Decimal of a Pound Sterling.



C H A P. II.

Of Extraction of the Square Root.

TO extract the Square Root of any whole or mixed Number.

D E F I N I T I O N.

A Square Number is produced by multiplying any Number by itself, as $4 \times 4 = 16$, the Square of 4, and $5 \times 5 = 25$, the Square of 5, &c.

The Roots are either simple or compound. The simple Roots with their Squares are exhibited in the following Table, which ought to be got by Heart :

A Table of Roots and Squares.

Rs.	1	2	3	4	5	6	7	8	9
Sqs.	1	4	9	16	25	36	49	64	81

Compound Roots are those which consist of more than one Figure, as $36 \times 36 = 1296$, the Square of 36, &c. Therefore to extract the Square Root of any Number is to find a Number that being multiplied by itself will produce the Number given. And in order to this observe the following Rules.

I. Put a Point over every second Figure (beginning at the Unit's Place from the right Hand to the left ; and as many Points or Periods as there are in the given Number, so many Figures will be in the Root.

II. See what is the nearest less Square Number in your first Period to the left Hand, which may be known by the foregoing Table of Roots and Squares, and whatever you find it to be, place the Root of it in the Quotient, and the Square of it under the first Period, and subtract it from that Period ; then to the Remainder bring down the next two Figures or Period.

III. To find what the next Figure of the Root will be, double the Root before found, and set it to the left Hand of the Remainder for a Divisor ; then see how often you can have that Divisor in the Remainder, augmented by one of the Figures last brought down ; and as often as you find it will go, place it in the Root, and also in the Unit's Place of the Divisor.

IV. Lastly, Multiply this augmented Divisor by the last Figure set in the Root, and set the Product under

Extraction of the Square Root.

under the Remainder, and subtract it from it; then bring down the Figures in the next Period, and add them to this last Remainder. Then again double the Root for a Divisor, and thus proceed to do till all the Periods are brought down; and if any Thing remains add two Cyphers to the Remainder and repeat the Work. Note, for every Pair of Cyphers you add, you will have one Decimal in the Root. When the given Number consists of whole Numbers and Decimals, the whole Numbers must be pointed as before directed. But the Decimals must be pointed the contrary Way, viz. every second Figure to the right Hand from the Unit's Place; and when there is an odd Decimal add a Cypher to make their Number even.

EXAMPLES.

What is the Square Root of 256?

OPERATION.

$$\begin{array}{r} 256 \div 16 = \text{Root.} \\ 1 \\ \hline 26 \overline{)156} \\ \underline{156} \\ 0 \end{array}$$

In the above Example there are two Points, the first on the Figure 2. The nearest less Square to 2 is 1, which also is the Root; therefore I place the 1 in the Root, and also under the Figure 2, which is the first Period; then I subtract the Figure 1 from 2 and there remains 1, to which I bring down the next Period 56; then I double the Root, viz. $1 \times 2 = 2$, which I set in the Divisor's Place; then I seek how often 2 will go in 15, which I find to be six Times; therefore I place 6 in the Root, and also in the Divisor's Place. Then I multiply my Divisor 26 by 6 the last Figure in the Root, and the Product 156 I subtract from the Remainder augmented by the last Period, and there remains nothing. So I find the Root of 256 to be 16; for $16 \times 16 = 256$.

The Way to prove the Work is to multiply the Root by itself, and the Product, with the Remainder, (if any) will be equal to the given Square Number, if right, otherwise not.

EXAMPLE II.

What is the Square Root of 282?

OPR-

10 Extraction of the Square Root.

OPERATION.

$$\begin{array}{r}
 \sqrt{2821679} = \text{Root.} \\
 \begin{array}{r}
 1 \\
 \hline
 26 \overline{)182} \\
 \underline{156} \\
 327 \overline{)2600} \\
 \underline{2289} \\
 3349 \overline{)31100} \\
 \underline{30141} \\
 \text{Remain } 959
 \end{array}
 \end{array}$$

EXAMPLE III.

What is the Square Root of 62650,9?

OPERATION.

$$\begin{array}{r}
 \sqrt{62650,90} (250,301 = \text{Root.} \\
 \begin{array}{r}
 4 \\
 \hline
 45 \overline{)226} \\
 \underline{225} \\
 5003 \overline{)15090} \\
 \underline{15009} \\
 500601 \overline{)810000} \\
 \underline{500601} \\
 309399 \text{ Remainder.}
 \end{array}
 \end{array}$$

When you cannot have your Divisor in your Dividend (without counting the Figure in the Unit's Place) then add a Cypher to the Root, and also to the Divisor, as in the last Example, and bring down the next Period, (if any) or add a Pair of Cyphers, and proceed according to the foregoing Rules. And in this Manner may the Square Root of any other whole or mixed Number be extracted.

II. To extract the Square Root of a Decimal Fraction.

THIS differs nothing from the extracting the Roots of whole Numbers, except in pricking off of the Roots; for as in whole Numbers we begin to prick off every second Figure from the right Hand to the left, so in Decimals we must begin at the left Hand, and prick off every second Figure to the right Hand, which is just the Reverse.

Notes,

Extraction of the Square Root. II

Note, If the Number of Figures are odd, always add a Cypher to the right Hand of the given Sum before you begin your Work, to make them even. A few Examples will make this plain.

EXAMPLE I.

What is the Square Root of ,00046225?

OPERATION.

,00046225,0215 = Root.

$$\begin{array}{r} 4 \\ \hline 41)062 \\ 41 \\ \hline 425)2125 \\ 2125 \\ \hline \end{array}$$

o Remainder.

What is the Square Root of ,06435?

EXAMPLE II.

OPERATION.

,064350,25367 = Root.

$$\begin{array}{r} 4 \\ \hline 45)243 \\ 225 \\ \hline 503)1850 \\ 1509 \\ \hline 5066)34100 \\ 30396 \\ \hline 50727)370400 \\ 355089 \\ \hline \end{array}$$

15311 Remainder.

EXAMPLE III.

What is the Square Root of ,35864?

OPE.

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OPERATION.

$.3586401598865 = \text{Root.}$

25

109) 1086

981

1188) 10540

9504

11968) 103600

95744

119766) 785600

718596

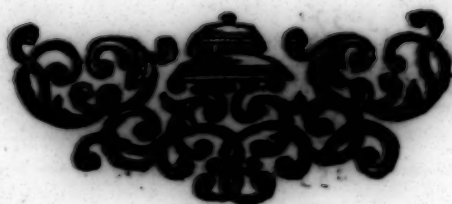
1197725) 6700400

5988625

711775 Remainder.

Thus may the Roots be found to what Exactness you please by constantly adding two Cyphers to the Remainder. But six Decimals are enough in most Cases, and even two are enough in many Cases.

If what has been said on this Head be perfectly understood, it will be no difficult Matter to extract the Square Root of any Number whatsoever; therefore I shall add no more Examples on this Head, but proceed to the next.



CHAP. III.

Of Extraction of the Cube Root.

DEFINITION.

I. **A** Cube Number is produced by multiplying any Number by itself, and again multiplying that Product by the same Number: That is, if you square any Number, and multiply that Square by its Root, the Product will be the Cube of that Number. Thus, $2 \times 2 = 4$, the Square of 2; and $4 \times 2 = 8$, the Cube of 2. Again, $5 \times 5 = 25$, the Square of 5; and $25 \times 5 = 125$, the Cube of 5; and so of any other.

II. The Roots are either simple or compound. The simple Roots are found by Inspection as in the following Table;

A Table of Roots and Cubes.

Rs.	1	2	3	4	5	6	7	8	9
Cs.	1	8	27	64	125	216	343	512	729

The compound Roots are such as consist of two or more Figures; as 12 is a compound Root, and $12 \times 12 = 144$, the Square of 12, and $144 \times 12 = 1728$, the Cube of 12; and so of any other Numbers.

To extract the Compound Cube Root of any Number given.

RULE.

1. **A**S in the Extraction of the Square Root put a Point over the Unit's Place of the given Number, and instead of putting the next Points over every second Figure put it over every third Figure towards the left Hand; and as many Points, or Periods, as there are, so many Figures will be in the Root. If there are any Decimals in the given Number, then prick off every third Decimal from the Unit's Place the contrary Way, ~~and~~ from the left to the right Hand; and when there happens to be but one or two Decimals in the given Number, then add one or two Cyphers to make up three Figures in order to complete the Period.

2. Find the nearest lesser Cube (by the foregoing Table) in the first Period towards the left Hand, and place the Root of it on the right Side of the crooked Line, and the Cube of it set underneath the first Period and subtract it from it; ~~and to the~~ **Remainder**

14 Of Extraction of the Cube Root.

Remainder bring down the next Period, which call the Resolvend.

3. To find the next Figure in the Root triple the Root, and set the Unit's Place of the triple Root under the Unit's Place of the Resolvend. Then square the Root and triple that Square, the Product of which set down as in Multiplication under the triple Root, *viz.* Units under the Ten's Place; then draw a Line and add these two Numbers together for a Divisor.

IV. Find how often you can have your Divisor in the Resolvend without counting the Unit's Place of the Resolvend. And as often as you can have your Divisor in that Part of the Resolvend set in the Root's Place, and observe the following Directions:

1. Cube the Figure last placed in the Root, and set the Unit's Place of that Cube under the Unit's Place of the Resolvend.

2. Square the same and multiply that Square by the triple Root before found, and set the Unit's Place of this Product under the Ten's Place of the Cube.

3. Multiply the triple Square of the Root by the last Figure placed in the Root, which Product set down under the Product of the Square and triple Root, placing Units under Tens, as before.

4. Draw a Line, and add these three Sums together and subtract it from the Resolvend, and to the Remainder bring down the next Period, (if any) or add three Cyphers, which will be a new Resolvend; to which, by the foregoing Rules, find a new Divisor, and thus proceed in all the Steps as before directed till all the Periods are brought down, or till you have gained as many Decimals in the Root as you shall find needful. The following Examples will make it more plain:

EXAMPLE I.

What is the Cube Root of 1728?

OPERATION.

1728 (12 = Root.

1 Cube of 1 subtract.

0728 Resolvend.

3 Triple of the Root 1.

3 Triple Square of the Root 1.

33 Divisor.

8 Cube of 2.

12 Triple of Root 1 by Square of 2.

6 Triple Square Root 1 by 2.

Sum 728 Subtract from Resolvend.

0 Remain.

In the foregoing Example I put a Point over the Figure 8 in the Unit's Place, and on the Figure 1, which

Extraction of the Cube Root. 15

which is the third Figure towards the left Hand; then the first Period is 1, and the nearest less Cube of 1 is 1, and 1 is also the Root, which I place in the Root's Place, and under the first Period; then 1 from 1 subtracted leaves nothing, to which I bring down the next Period 728, which is the Resolvend. The Root 1 tripled is 3, which I place under the Unit's Place of the Resolvend; and the Square of 1 is 1, which tripled is also 3, which I place in the Ten's Place under the triple Root. The Sum of these two Numbers is 33, which is the Divisor: Then I seek how often I can have 33 in 72, the two first Figures of my Resolvend, which I find to be twice: Therefore I set the Figure 2 in the Root's Place, then I cube the Figure 2, which is equal to 8, and set it under the Unit's Place of the Resolvend; then the Square of 2 = 4 I multiply by the triple Root 3, which = 12, and set it down under the Ten's Place of the Cube; and lastly I multiply the triple Square of the Root 1 by 12, which = 6, and place it under the Ten's Place of the last Product. The Sum of these three Numbers thus found = 728, I subtract from the Resolvend, and I find nothing remains, and the Root required = 12.

EXAMPLE II.

What is the Cube Root of 34645976?

OPERATION.

34645976	(326 = Root.
27	Cube of the Root 3.
7645	Resolvend.
9	Triple of the Root 3.
27	Triple Square of the Root 3.
279	Divisor.
8	Cube of 2.
36	Square of 2 x by the Triple Root.
54	Triple Square of the Root x by 2.
Sum 5768	Subtract from the Resolvend.
1877976	Resolvend.
96	Triple Root 32.
3072	Triple Square of the Root 32.
30816	Divisor.
216	Cube of 6.
3456	Square of 6 x into the Triple Root.
18432	Triple Square of the Root x by 6.
Sum 1877976	Subtract from last Resolvend.
Remains 0	

EXAM.

16 *Extraction of the Cube Root.*

EXAMPLE III.

What is the Cube Root of 47,5?

OPERATION.

47,500(3,62 = Root.

27

20500 Resolvend.

9

27

279 Divisor.

216

324

162

19656 Subtrahend.

844000 Resolvend.

108

3888

38988 Divisor.

8

432

7776

781928 Subtrahend.

Rem. 62072

Thus I find the Root of 47,5 = 3,62, and something more. If you have a Mind to have the Root nearer the Truth, you may, by adding of three Cyphers to the Remainder for every Decimal you gain in the Root, come to what Exactness you please. Here observe that it will be necessary to square the Root upon a Piece of waste Paper when it becomes large, and also to find your Subtrahend in the same Manner, before you enter it in its proper Place; for then, if you find it to exceed your particular Resolvend, you may place a less Figure in the Root, and work again afresh for a new Subtrahend, which must not exceed the Resolvend.

The Way to prove your Work is to multiply the Root by itself, and that Product by the Root; which Sum, with the Remainder, will be equal to the given whole Number if right, otherwise not.

II. Of the Use of the Cube Root.

THE chief Use of the Cube Root is to discover the Proportion of Lines to Solids, and of Solids to Lines; for like Solids, such as Globes, Cylinders, Cubes, &c. are in triplicate proportion to their like Solids: Therefore, as the Cube of the Diameter of any Solid is to its Weight, or Solidity, so is the Cube of the Diameter of another like Solid to the Weight or Solidity thereof.

EXAMPLE I.

If an iron Bullet of four Inches Diameter weigh 9 Pounds, What will an iron Bullet of 8 Inches Diameter weigh?

R U L E.

Multiply the Cube of the third Term by the second Term, and divide the Product by the Cube of the first term, and the Quotient will be the Answer.

O P E R A T I O N.

Cube. Wt.	Cube. Wt.
As 64 : 9 :: 512 : 72 = Answer.	
9	
—	
64)4608	
448	
—	
128	
128	
—	
0	

EXAMPLE II.

If a Fathom of Rope of 10 Inches Circumference weigh 17 lb. How much will a Fathom weigh of 8 Inches Circumference?

O P E R A T I O N.

Cube of 10	lb. C. of 8.	lb.
As 1000 : 17 :: 512 : 8 $\frac{704}{1000}$ = Answer.		
17		
—		
3584		
512		
—		
1,000)8,704 = Remainder.		

C

Another

ANOTHER EXAMPLE.

Having the Dimensions of a conical, or other Vessel given, with the Content thereof, and I wanted to make another that should contain more or less Gallons of a similar Form.

R U L E.

To solve such like Questions there must be three Proportions wrought: 1. As the Content of the given Vessel is to the Cube of the given Diameter at the Top, so is the Content of the Vessel sought to the Cube of its Top Diameter.

II. As the Content of the given Vessel is to the Cube of its Bottom Diameter, so are the Contents of that sought to the Cube of the required Diameter.

III. As the Content of the given Vessel is to the Cube of its Depth, so is the Content of the Vessel sought to the Cube of the Depth sought. Then, if you extract the Cube Root from these three Numbers, thus found, you will have the several Dimensions required.

FOR EXAMPLE.

Suppose a conical Tun of the following Dimensions and Content, *viz.* Top Diameter = 30 Inches, Bottom Diameter = 40 Inches, Depth = 20 Inches, and the Content = 68,5 Ale Gallons, and it were required to make another of the same Form, that would contain 90 Gallons, What must its Dimensions be?

1. Proportion for the Top Diameter.

Cont.	Cube of T.	Cont.	Cube of T.
	Diameter.		Diameter.
As 68,5	: 27000	:: 90	: 35474.452.

2. Proportion for the Bottom Diameter.

Gallons.	Cube of B.	Gallons.	Cube of B. Dia-
	Diameter.		meter.
As 68,5	: 64000	:: 90	: 84087.576.

3. Proportion for the Depth.

Gallons.	Cube of Depth,	Gall.	Cube of Depth.
As 68,5	: 8000	:: 90	: 10510.948.

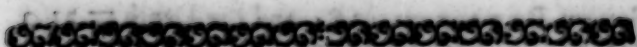
By extracting the Cube Roots of the fourth Terms of each of the three foregoing Proportions, I find the

Of the Uses of the Cube Root. 19

the Top Diameter required = 32,8 Inches; the Bottom Diameter = 43,8; and the Depth = 21,9 Inches; the Dimensions required.

But such Kind of Questions as these are solved with a vast deal less trouble by those sliding Rules which have the Line π upon them; for at once setting of the Rule for each Proportion, all the required Dimensions are found without any more Trouble. On account of the Excellency of the Cube Line upon the Rule for working any Question in triplicate Proportion, I have given Rules for extracting the Cube Root by the Pen in this Book, in order to enable the young Gager to do it the better by the Rule.





CH A P. IV.

A Description of the Sliding Rule, and of the several Lines upon it, with Directions how to find any Number thereon; together with the Application of those Lines to Multiplication, Division, the Rule of Three, &c.

A R T. I.

A Description of the Rule.

THIS Instrument is commonly made of Box or Ivory, of divers Lengths, viz. 6, 9, 12, 18 Inches long, and sometimes longer: But that is not material, because they are all made after one and the same Manner, with this only Difference, the longer the Lines are, the more Subdivisions they will admit of.

What I shall here treat of is the common Rule of 12 Inches long, which if thoroughly understood, that of any other Length will be easily known.

A R T. II.

An Explanation of the Lines on the Rule.

1. **O**F the Lines on the first Face or Side of the Rule: The first Line on this Face is that on the Rule which is distinguished by the Letter A, and is commonly known by the Name of Gunter's Line, it being a Line of Numbers in a geometrical Proportion, consisting of two Radius's, and is numbered from the left to the right Hand, with the Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. On this Line are put several Brass Center Pins or Dots: The first is placed at 2150.42 in the first Radius, and marked *m* 2, signifying the cubic Inches in a Bushel of Malt; the second is at 282 in the same Radius, and marked *a* 6, signifying the Number of cubic Inches in a Gallon of Ale; the like Marks and Dots are again repeated at the same Numbers in the second Radius.

2. The second Line on this Face is that on the Slide, and is known by the Letter *a*, which is also a Line of Numbers, of the same Length, and divided in the same Manner, as the Line A, when Unity at the End of one is applied to the End of the other, all the Figures of the

are parallel to those of the other, and each Division and Subdivision of the one are collateral to those of the other. On this Line are also placed several Brass Pins or Dots: The first is at 231, mark'd w c in the first Radius, signifying the cubic Inches in a Gallon of Wine; the second is at 3,14, and mark'd c, signifying the Circumference of a Circle whose Diameter is Unity, which Marks are again repeated in the second Radius; the third Dot is placed at 707, and mark'd s, signifying the Side of the greatest square inscribed in a Circle whose Diameter is Unity; the fourth is at 886, and mark'd se, signifying the Side of a Square equal in Area to that of a Circle whose Diameter is Unity.

3. The third Line on this Face is that on the Rule called the Malt Line, and known by the Letters m d, signifying Malt Depth, which is also a double Line of Numbers of the same Length, and divided in the same Manner, as the two former, but is broken and inverted, beginning at 2150,42, and is numbered from the left to the right Hand inversely, viz. 2, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 9, 8, 7, 6, 5, 4, 3, and ends at 2150,42, where it began.

Of the Lines on the second Face or Side of the Rule.

1. **T**HE first Line on this Side is put on the Rule, and known by the Letter n, which is a Line of Numbers of a single Radius, and is numbered on from 1 at the Beginning to 10 at the End. This Line of Numbers of a single Radius is just the same Length with the Lines a n, and m d before described. On this Line also are placed certain Gage Points; as, at 1715 is mark'd v c, signifying the circular Gage Point for Wine Gallons, at 18,05 is mark'd a 4, signifying the circular Gage Point for Ale Gallons, at 46,37 is mark'd m 1, signifying the Square Gage Point for Malt Bushels, at 52,38 is mark'd v a, signifying the round or circular Gage Point for Malt Bushels, and at 6,32 is mark'd t p, signifying the circular Gage Point for a Peck of No. Tallow.

Now, On this Line may all the other Gage Points mentioned in Page 44, be found, and they are not pointed out by Letters, and they are those before mentioned.

2. The second Line on this Side is that put upon the Slide, and known by the Letter c, which is a Line of Numbers of a double Radius, and is Length with the Line a, and sits to the Line a and n, and it is divided exactly in the same Manner as the Lines a and n.

When the Lines n and c are set for unity at both Ends, the Unity at the Beginning of the one is

Unity at the Beginning of the other; then the Line *c* represents the Squares, and the Line *d* the Roots of those Squares.

3. The third and fourth Lines on this Face are two Lines of Segments for ullaging of Casks, and are set on the Rule; one of which is mark'd *S^u*, that is, Segments lying, and the other *S^t*, Segments standing; and they are both alike numbered from the left Hand to the right, with the Figures 1, 2, 3, &c. to 10, and from 10, 20, 30, 40, to 100.

When the Line *e* is put upon the Rule, it is placed here in the Room of the two Lines of Segments, and these Segments are put on one of the Edges of the Rule with a Line of Numbers of double Radius to slide between them; and, in this Case it will be proper to have the Line *b* upon the Slide, and the Line *c* in *d*'s Place on the Rule, so as that the Line *b* may slide between the Lines *c* the Squares, and *e* the Cubes, for the Line *e* is a triple Line of Numbers, consisting of three Radius's, and is equal in Length to both the single and double Lines of Numbers; and as those Numbers on *c* are Squares to those on *d*, so are the Numbers on the Line *e* Cubes to those on *d*; so that if the Line *b* on the Slide be set even at both Ends with the Lines *c* and *e*, you have a Table of Cubes, Squares, and their Roots, viz. on *e* are the Cubes, on *c* are the Squares, and on *d* the Roots.

Of the Lines on the third Face or Edge of the Rule.

1. **T**HE first Line on this Face or Edge is a Line of Inches decimally divided, and numbered on from 1, 2, 3 to 12.

2. The second Line on this Edge is that mark'd *Spheroid*, and is number'd on from 1, 2, 3, to 7, which is a Line of Differences for reducing the first Variety of Casks to a mean Diameter.

3. The third Line on this Edge is that mark'd *2^d Variety*, and is number'd on from 1, 2, to 6; which is also a Line of Differences for reducing the second Variety of Casks to a mean Diameter.

4. The fourth Line on this Edge is mark'd *3^d Variety*, and is likewise a Line of Differences for reducing the third Variety of Casks to a mean Diameter.

Of the Lines on the fourth Face or Edge of the Rule.

1. **T**HE first Line on this Edge is mark'd π , signifying Foot Measure, and is number'd on from 1, 2, 3, to 10; which is for reducing of Inches, &c. to the decimal Parts of a Foot.

2. The second Line on this Edge is a Line of Inches decimally divided, and numbered on from 1, 2, 3, to 12.

3. The last Line on this Edge is that mark'd π 0, signifying the Frustrum of two Cones, and is numbered on from 1, 2, 3, to 6, which is a Line of Differences for reducing the fourth Variety of Casks to a mean Diameter.

These are all the Lines that are usually put on the outside Faces of the Rule: On the Inside, or underneath the Slides, there are generally put a Line of Inches, with their respective Areas in Ale Gallons at those several Diameters; but they being of so little Use in Practice, I shall take no more Notice of them: Besides, I think their Place might be supplied with something that would be of much more Service.

Thus I have described and explained all the Lines that are usually put on the Sliding Rule, and in the next Place intend to shew their Use.

But here I would observe that all these Lines are put on the *Brown's* Rule, except the Lines of Varieties, whose Place may be supplied with proper Factors that would answer the same End, and may be used in every Respect as a Sliding Rule, and even with more Advantage in some Cases, than by Means of a new Contrivance the *Calks* Line of Line 2 on the *Brown's* Rule may be made so as to slide by the Line π , or the Roots, which Line is not very common to be met with on the Sliding Rule.

A R T. II.

Of Numeration by the Line of Numbers.

1. **T**HE Line of Numbers is divided into ten unequal Parts, called *Orders*, beginning at the right hand, and ending at Unity; and each of these Orders are again divided into ten other unequal Parts,

called Tenths; and each of those Tenths are subdivided or supposed to be divided into ten other Parts, called Centesims, according as the Length of the Line, or Radius will bear.

1. Every Tenth from 1 to 2 on the Line $\mathfrak{D}$ is actually divided into ten Centesims: But from 2 to 3 the Tenths are only divided into five Parts; so there each Division contains two Centesims. From 3 to the End of the Line $\mathfrak{D}$ each Tenth is divided but into two Parts, and each of these Parts contain five Centesims.

2. In the double Lines of Number, *viz.* the Lines A, B, and C, the Tenths from 1 to 2 will admit of only five Divisions, each of which Divisions contain two Centesims. From 2 to 5 each Tenth is divided into but two Parts, each Part containing five Centesims; and from 5 to 10 the Tenths will not admit of any Division at all, without crowding one another; therefore the Centesims must be guessed at in those Places.

3. The Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, are all arbitrary Points, and may be conceived to represent so many entire Units, Tens, Hundreds, or Thousands.

Or they may represent so many tenth, hundredth, or thousand Parts of an Unit.

If the Figure 1 at the Beginning of the Line represent one Unit, then will the Figures 2, 3, 4, 5, 6, 7, 8, 9, represent so many Units, and the Tenths in each of those Primes will be the tenth Part of Unity; the Centesims in each of those Tenths will be the hundredth Parts of Unity.

Or if 1 at the Beginning of the Line represent ten Units, then will each Figure forward towards the right Hand represent ten Times as many Units as the Figure expresses. As for Example, if the Figure 1 at the Beginning of the Line represents ten Units, then will the Figure 2 represent 20 Units, the Figure 3 30 Units, and the Figure 4 40 Units, the Figure 5 will represent 50 Units, &c. till you come to 1 or 10 at the End of the Radius, which will represent 100 Units.

Or the Figure 1 at the Beginning of the Line may represent one-tenth Part of an Unit, then each Prime towards the right Hand will represent so many tenth Parts of Unity, and each Tenth in those Primes will be hundredth Parts of Unity, and each Centesim in those Tenths will be the thousandth Parts of Unity.

Of the Sliding Rule.

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That this may be the better conceived, I shall subjoin the following Schemes:

1. For the single Radius, or Line D.

From 1 to 2 the Tenths are divided into ten Parts, called Centesims: Therefore

If the figure 1 at the beginning of the line represent	{	10.	1.	.1	}	the larger Divisions will represent	{	10.	1.	.01	}	and the smaller divisions will represent	{	1.	.01	.001	}

From 2 to 5 the Tenths are divided into five Parts, each Part containing two Centesims: Therefore

If the Figure 2 represent	{	200.	20.	2.	.2	}	the larger Divisions will represent	{	10.	1.	.01	}	and the smaller Divisions will represent	{	2.	.02	.002	}

From 3 to the End of the Line D, the Tenths are only divided into two Parts, each containing five Centesims: Therefore

If the Figure 3 represent	{	300.	30.	3.	.3	}	the larger Divisions will represent	{	10.	1.	.01	}	and the smaller Divisions will represent	{	5.	.05	.005	}

2dly, For the Lines A, B, and C, of double Radius.

From 1 to 2 the Tenths are divided into five Parts, each Part containing two Centesims: Therefore

If the Figure 1 at the beginning of the line represent	{	100.	10.	1.	}	the larger Divisions will represent	{	10.	1.	}	and the smaller Divi- sions will represent	{	1.	.01	.002	}

From

From 2 to 5 the Tenths are divided into two Parts, each Part containing five Centesims. Therefore

If the Figure 2 represent	{	20.	2.	.2	}	{	10.	.01	}	{	5.	.05	}	{	.005	}
the larger Divisions will represent																
and the smaller Divisions will represent																

From 5 to 10 the Primes are only divided into ten Parts, so the Centesims must be guessed at:

Therefore if the Figure 5 represent	{	500.	50.	5.	}	{	the larger Divisions, or Tenths, will represent	{	10.	.1	}	{	.01	}

3dly, For the Line 2 of triple Radius.

From 1 to 4 the Tenths are divided into two Parts, each Part containing five Centesims: Therefore

If 1 at the Beginning of the Line repre- sent	{	100.	{	10.	{	.1	}	{	10.	{	.01	}	{	5.	{	.05	{	.005	}
the larger Divisions will represent																			
and the smaller Divi- sions will represent																			

From 4 to the End of the Radius, the Tenths, for Want of Room, are not divided at all:

Therefore if the Figure 4 represent	{	400.	40.	4.	}	{	10.	.1	}	{	.01	}

By the foregoing Schemes it will be an easy Matter to conceive what the Tenths and Centesims represent, when the Value of the Primes are fix'd.

Here

Here it will not be amiss to observe, 1. Of whatever Denomination 1 at the Beginning of the single Radius or Line *b* is of, the 10 at the End thereof will be ten Times as many.

2. Of whatever Denomination the 1 at the Beginning of the Lines of double Radius, viz. the Lines *A*, *B*, and *C* are of, the 1 in the Middle is ten Times as many, and the 10 at the End thereof is a hundred Times as many.

3. Of whatever Denomination the 1 at the Beginning of the triple Line or Line *a* is of, that the next 1 towards the right Hand is ten Times as many, and the third 1 or 10 is a hundred Times as many, and the last 1 or 100 is a thousand Times as many.

By what has been said I imagine that it will be no difficult Matter to find the Point upon any of the Lines where any given Number is represented. As for Example, suppose 17,15 were required to be found upon the Line *b*.

For the first Figure (viz. 1) count 1 at the Beginning of the Line *b* for 10, for the second Figure 7 count 7 of the following larger Divisions, or Tenths, for seven Units, which is the Point where 17 stands; then from this Point forward count one Centesim for the third Figure 1, and for the last Figure 5 count half the next Centesim; and at that Point you will find the Dot mark'd with *w* o, which represents 17,15, which was required.

If it had been required to find 1715 1,715 171,5 or 1715, they would all be found at the same Point: But then the Figure 1 at the Beginning of the Line would be conceived of a different Value in finding each of the above five Numbers, viz. 1. 10. 100. 1000.

By the same Rule will the Numbers 18,95 be found at the Point *A* G. 46.37 at *M* S. 52.32 at *M* A. and 6.32 at *tp*.

And in this Manner is the Point that represents any other Number found either upon this Line or any of the others.

But it will be proper to observe the following Directions:

1. If the given Number consist of three Figures, the middle being a Cypher, then you must find it in the first Tenth following the Prime of that Number. For Example, let the given Number be 406, the first Figure 4 = 400 find at the fourth Prime:

the

From 2 to 5 the Tenths are divided into two Parts, each Part containing five Centesims: Therefore

If the Figure 2 represent	{	20.	2.	.2	}	the larger Divisions will represent	{	10.	1.	.1	}	and the smaller Divisions will represent	{	5.	.5	.05	.005	}

From 5 to 10 the Primes are only divided into ten Parts, so the Centesims must be guessed at:

Therefore if the Figure 5 represent	{	500.	50.	5.	.5	}	the larger Divisions, or Tenths, will represent	{	10.	1.	.1	}	{	.01	}

3dly, For the Line 2 of triple Radius.

From 1 to 4 the Tenths are divided into two Parts, each Part containing five Centesims: Therefore

If 1 at the Beginning of the Line repre- sent	{	100.	10.	1.	}	the larger Divisions will represent	{	10.	1.	}	and the smaller Divi- sions will represent	{	5.	.5	.05	.005	}

From 4 to the End of the Radius, the Tenths, for Want of Room, are not divided at all:

Therefore if the Figure 4 represent	{	400.	40.	4.	.4	}	then the larger Divisions, or Tenths, will represent	{	10.	1.	.1	}	{	.01	}

By the foregoing Schemes it will be an easy Matter to conceive what the Tenths and Centesims represent, when the Value of the Primes are fix'd.

Here

Here it will not be amiss to observe, 1. Of whatsoever Denomination 1 at the Beginning of the single Radius or Line D is of, the 10 at the End thereof will be ten Times as many.

2. Of whatsoever Denomination the 1 at the Beginning of the Lines of double Radius, viz. the Lines A, B, and C are of, the 1 in the Middle is ten Times as many, and the 10 at the End thereof is a hundred Times as many.

3. Of whatsoever Denomination the 1 at the Beginning of the triple Line or Line E is of, that the next 1 towards the right Hand is ten Times as many, and the third 1 or 10 is a hundred Times as many, and the last 1 or 100 is a thousand Times as many.

By what has been said I imagine that it will be no difficult Matter to find the Point upon any of the Lines where any given Number is represented. As for Example, suppose 17,15 were required to be found upon the Line D.

For the first Figure (viz. 1) count 1 at the Beginning of the Line D for 10, for the second Figure 7 count 7 of the following larger Divisions, or Tenths, for seven Units, which is the Point where 17 stands; then from this Point forward count one Centesm for the third Figure 1, and for the last Figure 5 count half the next Centesm; and at that Point you will find the Dot mark'd with w o, which represents 17,15, which was required.

If it had been required to find 1715 1,715 171,5 or 1715, they would all be found at the same Point: But then the Figure 1 at the Beginning of the Line would be conceived of a different Value in finding each of the above five Numbers, viz. 1. 10. 100. 1000.

By the same Rule will the Numbers 18,95 be found at the Point A G. 46.37 at M S. 52.32 at M A. and 6.32 at tp.

And in this Manner is the Point that represents any other Number found either upon this Line or any of the others.

But it will be proper to observe the following Directions:

1. If the given Number consist of three Figures, the middle being a Cypher, then you must find it in the first Tenth following the Prime of that Number. For Example, let the given Number be 406, the first Figure 4 = 400 find at the fourth Prime: the

the second Figure being a Cypher, count no Tenths : But for the last Figure 6 count six Centesims, which is a little to the right Hand of the lesser Division in that Tenth ; and that is the Point where 406 is represented. If the Number 404 had been to be found, it would have been found the same Distance from the lesser Division, but on the other Side towards the Prime.

2. If the given Number consists of four Places of Figures, having two Cyphers in the Middle, then it will be found near the Prime unto which it belongs, *viz.* between the Prime and the next Centesim following towards the right Hand. As for Example, If the Number 2005 were to be found, the first Figure 2 = 2000 is found at the Beginning of the second Prime, and because there are Cyphers in the second and third Places, you must not count any of the Tenths or Centesims, but for the last Figure 5 count half of the first Centesim towards the right Hand, and that will be the Point where 2005 is represented.

3. If the given Number consist of 2, 3, or 4 Places of Figures, and all of them Cyphers, except the first, then they are all found at the Beginning of the Prime unto which they belong ; so that if the Numbers were .003, .03, 30, 300, 3000, they are all found at the same Point, *viz.* at the Beginning of the third Prime : Or if the Numbers were, .004, .04, 40, 400, or 4000, they are all found at the same Point, *viz.* at the Beginning of the fourth Prime ; and so of any other Number. Therefore if what has been already said concerning Numeration on the Rule be duly considered, and thoroughly understood, it will be sufficient for the finding the Point on the Rule where any Number whatsoever is represented, without adding any more Examples ; So I shall proceed, in the next Place, to shew the Application of those Lines to Multiplication.

A R T. III.

Of Multiplication by the Lines A and B.

WHEN two Numbers are given to be multiplied together, whether they be whole Numbers or decimal Fractions. The Proportion is,

As Unity on *B* is to the Multiplier on *A*, So is the Multiplicand on *B* to the Product on *A*.

Or,

As Unity on *A* is to the Multiplier on *B*, So is the Multiplicand on *A* to the Product on *B*.

EXAMPLE I.

Let the Product of 8 by 6 be sought.

OPERATION.

Set 1 on B to 6 on A, and against 8 on B is 48 upon A, the Product required.

Or,

As 1 on A is to 6 on B, so is 8 on A to 48 on B, the same as before.

Thus you see the first Term or Unity may be taken upon either of the Lines A or B, provided the third Term be taken upon the same Line as the first, and the fourth Term on the same Line as the second.

EXAMPLE II.

What is the Product of 65 by 18?

OPERATION.

Set 1 on B to 18 on A, and against 65 on B is 1170 on A, the Product required.

EXAMPLE III.

What is the Product of 76 by 13?

OPERATION.

Set 1 on B to 13 on A, and against 76 on B is 988 on A, the Product required.

And in this Manner may the Product of any other two Numbers be found: But in order to know how many Figures the Product will consist of, observe the following Directions:

1. If the first Figures of the Product are less than the least of the two Factors, the Product will have as many Places as both Factors.
2. If the first Figures of the Product are equal or exceed the least of the Factors, then the Product will have one Place less than the two Factors.

These Rules are manifest by the foregoing Examples; for in the first Example the least of the Factors is 6, and the first Figure of the Product is 4, which is less than 6: Therefore the Product is 48, that is, it consists of two Places of Figures equal

equal to the Number of Places of both Factors. And in the second Example likewise the least of the Factors is 18, and the two first Figures of the Product are 11, which being less than 18, therefore the Product consists of four Places of Figures, and is = 1170, equal to the Number of Places of both Factors. But in the third Example the least of the Factors is 13, and the two first Figures of the Product are 98, which is greater than 13: Therefore the Product consists of but three Places of Figures, and is = 988, that is, one Place less than both Factors.

The next Thing to be considered is, to know the Value of the Products, whether they be whole Numbers, mixt Numbers, or decimal Fractions; as for Instance, in the last Example, where the Product was found to be 988, and not 98.8, or 9.88, or .988, as is plain by the following Table; for set 1 on B to 13 on A.

Then against	{ 2 3 4 5 6 7 7.6 }	upon B is	{ 26 39 52 65 78 91 98.8 }	upon A.

And if the Numbers on B be multiplied by 10, those also upon A will be multiplied by 10, that is, if 2 be supposed to be 20, then will 26 be equal to 260; and if 3 be supposed to be 30, then will 39 = 390, and so of all the rest; and 7.6 will become 76, and 98.8 will be = 988; which is the true Product, as before found. If 76 had been a Decimal, viz. .76, then the Product would have been .988; or if it had been .076, then the Product would have been .0988.

When one of the Factors is a whole or mixt Number and the other a decimal Fraction, then set Unity on the Middle of B to the whole or mixt Number on A, and seek the Fraction toward the left Hand on B, because it is less than Unity, and against it is the Product on A, which is always less than the given whole or mixt Number. For Example,

What is the Product of 45.3 by .7?

OPERATION.

Set 1 on the Middle of B to 45.3 on A, and against 7 on B is 31.71 on A, the Product required. I might here add more Examples: But I think the foregoing Examples and Directions are sufficient for any Variety that can occur: Therefore I shall proceed to the next Article of Division by Lines.

A R T. IV.

Of Division by the Lines A and B.

WHEN one Number is given to be divided by another, whether whole Numbers, mixt Numbers, or decimal Fractions, the Proportion is,

As the Divisor upon A is to Unity on B, so is the Dividend on A to the Quotient on B.

Or,

As the Divisor on B is to Unity on A, so is the Dividend on B to the Quotient on A.

E X A M P L E I.

What is the Quotient of 48 divided by 6?

O P E R A T I O N.

Set 6 upon A to Unity on B, then against 48 on A is 8 upon B, the Quotient required.

Or,

Set 6 upon B to Unity on A, then against 48 on B is 8 on A, the same as before.

E X A M P L E II.

What is the Quotient of 1170 by 18?

O P E R A T I O N.

Set 18 upon A to Unity on B, then against 1170 on A is 65 upon B, the Quotient required.

E X A M P L E III.

What is the Quotient of 988 divided by 13?

O P E R A T I O N.

Set 13 on A to Unity on B, then against 988 on A will be 76 on B, the Quotient required.

Any other Quotient may be found in like Manner: But in order to discover how many Figures the Quotient will consist of, observe the following Rule:

The Quotient must consist of as many Figures as there is Difference of Figures between the Dividend and the Divisor, except when the first Figure of the Dividend

Dividend are equal to, or more than the Divisor; and then it must have one Place more.

This will appear from the foregoing Examples; for in the first and second Examples the Divisors were greater than the first Figures of the Dividends, viz. 6 is greater than 2, and 18 is more than 11, the two first Figures of the Dividend in the second Example.

Therefore the Quotients in both Examples are equal to the Difference of the Number of Figures in the Divisor and the Dividend: But in the third Example the two first Figures of the Dividend = 98, are greater than the Divisor = 13: Therefore the Quotient consists of two Figures, that is, of one Place more than the Difference between the Divisor and the Dividend. And thus it may be determined, in all Cases, of how many Places of Figures the Quotient shall consist.

And the Value of the Quotient will appear in this Manner: For Instance, when 13 on A is set to Unity on B, as in the last Example,

$$\text{Then against } \left\{ \begin{array}{c} 26 \\ 39 \\ 52 \\ 65 \\ 78 \\ 91 \\ 98.8 \end{array} \right\} \text{ upon A is } \left\{ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 7.6 \end{array} \right\} \text{ upon B.}$$

And if those Numbers upon A are supposed to be multiplied by 10, those also on B will be multiplied by the same; for if we count 26 = 260, then will 2 = 20; if 39 be called 390, then will 3 = 30; if 52 be reckoned 520, then will 4 = 40, &c. till you come to 98.8, which multiplied by 10 = 988, and then 7.6 will become 76 = the true Quotient before found. From hence it is manifest, that at once setting the Rule both Multiplication and Division are performed; for the last Example is the Converse of the third Example in Multiplication, and when the Rule is so set, the Lines A and B are in Effect Tables of Products and Quotients; for if 13 be taken as a Multiplier, set Unity on B to 13 on A, then against any Multiplicand on B you have the Product upon A, as against 2 is 26, against 3 is 39, against 4 is 52, &c. Again, if you suppose 13 a Divisor, as the Rule now stands against any Dividend on A, is the Quotient on B, as against 26 on A is 2 on B, against 39 on A is 3 on B, against 52 on A is 4 on B, &c. By which it is evident, that Multiplication and Division are but the Converse of one another; so that if the one is understood, the other cannot be unknown.

The foregoing Rule and Examples, I imagine

will be sufficient to give the young Gager an Insight into this Matter: Therefore I shall add no more on this Head, but proceed to the next.

A R T. V.

Of the Single Rule of Three direct by the Lines A and B.

IN the Single Rule of Three direct there are always three Numbers given to find a fourth in direct Proportion.

The Proportion is,

As the first Term upon A is to the second Term upon B, so is the third Term upon A to the fourth Term on B.

Or,

Set the first Term upon B to the second on A, and against the third Term on B is the fourth Term upon A.

Note, the first and third Terms must always be taken on the same Line, viz. on either A or B, and the second and fourth Terms must be found on the other.

EXAMPLE I.

If 8 Yards of Cloth cost 48s. what will 18 Yards cost at that Rate?

OPERATION.

ON A. ON B. ON A. ON B.
As 8 : 48 :: 18 : 108 = Answer.
Yards. Shilla. Yards. Shilla.

Or,

ON B. ON A. ON B. ON A.
As 8 : 48 :: 18 : 108 = the same as before.

By which it appears that it matters not on which Line you take the first Term, provided you take the third Term on the same Line, and the second and fourth Term on the other Line.

EXAMPLE II.

If 3 Gallons of Ale cost 2s. what will 36 Gallons cost at that Rate?

D

OPE.

Of the Sliding Rule.

OPERATION.

ON B.	ON A.	ON B.	ON A.	
As 3	: 2	:: 36	: 24	= Answer.
Galls.	Shills.	Galls.	Shills.	

EXAMPLE III.

What is the Weight of 30 Rods of Candles of 12 to the Pound, when there are 24 Candles on each Rod?

OPERATION.

ON B.	ON A.	ON B.	ON A.	
As 12	: 30	:: 24	: 60	= Answer.
Numb. of Candles.	Rods.	Numb. of Candles.	Weight.	

More Examples might be adduced: But these being well understood will be sufficient to work any Proportion whatsoever.

ART. VI.

Of Inverse Proportion, or the Single Rule of Three indirect by the Lines A and B.

INverse Proportion is when there are three Numbers given to find a fourth Number that shall bear the same Proportion to the second Term as the first Term does to the third, which is known by the Nature of the Question, as when more requires less, or less requires more.

EXAMPLE I.

If 8 Men can do a Piece of Work in 20 Days, how long will 4 Men take to perform the same?

PROPORTION.

	1	2	3	4	Terms.
As 8	: 20	:: 4	: 40		

Operation by the Sliding Rule.

ON B.	ON A.	ON B.	ON A.	
As 4	: 8	:: 20	: 40	= Answer.
Men.	Men.	Days.	Days.	

By the Nature of this Question it is evident, that it is inverse Proportion; for less Men must require more Time to perform the same Piece of Work.

EXAMPLE II.

If 4 Men can do a Piece of Work in 40 Days, how many Men will do the same Work in 20 Days?

PRO-

P R O P O R T I O N .

1 2 3 4 Terms.
As 40 : 4 :: 20 : 8 = Answer.

By the Rule.

ON B. ON A. ON B. ON A.
As 20 : 40 :: 4 : 8 = the same as above.

Here it is also evident, that less Time requires more Men to perform the same Work in. And in this Manner are such Kind of Questions discovered to be in the inverse Rule of Proportion.

A R T. VII.

To reduce a Vulgar Fraction to its equivalent Decimal Expression.

The Proportion is

AS the Denominator upon A is to Unity on B, so is the Numerator on A to the Decimal required on B.

E X A M P L E I.

Reduce $\frac{1}{4}$ to a Decimal Fraction.

O P E R A T I O N .

ON A. ON B. ON A. ON B.
As 4 : 1 :: 1 : .25 = Decimal sought.
Denom. Unity. Num. Decimal.

E X A M P L E II.

Reduce $\frac{2}{11}$ to a decimal Fraction.

O P E R A T I O N .

ON A. ON B. ON A. ON B.
As 11 : 1 :: 2 : .18 = Decimal sought.
Denom. Unity. Num. Decimal.

E X A M P L E III.

Reduce $\frac{18}{28}$ to a decimal Fraction.

O P E R A T I O N .

ON A. ON B. ON A. ON B.
As 28 : 1 :: 18 : .643 fere = Dec. sought.
Denom. Unity. Num. Decimal.

And in like Manner may any other vulgar Fraction whatsoever be reduced to its equivalent decimal Expression.

A R T. VIII.

To find a Factor to a Divisor that shall perform the same by Multiplication as the Divisor would do by Division.

The Proportion is

As the Divisor given is to Unity, so is Unity to the Factor required.

E X A M P L E I.

Suppose 25 to be the Divisor, what will be the Factor to that Number?

O P E R A T I O N.

ON A. ON B. ON A. ON B.

As 25 : 1 :: 1 : ,04 = Factor sought.
Divisor. Unity. Unity. Factor.

P R O O F.

Let 25 be divided by 25, the Divisor, and multiplied by ,04, the Factor above found.

By Division.

By Multiplication.

$$\begin{array}{r} 25 \overline{) 25} \quad (21 = \text{Quotient.} \\ \underline{50} \\ 25 \\ \underline{25} \\ 0 \end{array}$$

$$\begin{array}{r} 5,25 \\ \underline{04} \\ 21,00 = \text{Product.} \end{array}$$

By this you see, that the same Thing may be effected by Multiplication as by Division; for the Quotient and the Product are both equal.

E X A M P L E II.

What will the Factor to 80 be?

O P E R A T I O N.

ON A. ON B. ON A. ON B.

As 80 : 1 :: 1 : ,0125 = Factor sought.
Div. Unity. Unity. Factor.

This being so easy, it would be needless to add any more Examples.

A R T.

A R T. IX.

Having a Factor given to find a Divisor.

THIS is the Converse to the last Article :
Therefore the Proportion is

As the Factor is to Unity, so is Unity to the Divisor required.

E X A M P L E I.

Let ,04 be the Factor given to find a Divisor.

O P E R A T I O N.

ON A. ON B. ON A. ON B.

As ,04 : 1 :: 1 : 25 = Divisor sought.
Factor. Unity. Unity. Divisor.

E X A M P L E II.

Let ,0125 be the given Factor, what will be the Divisor ?

O P E R A T I O N.

ON A. ON B. ON A. ON B.

As ,0125 : 1 :: 1 : 80 = Divisor sought.
Factor. Unity. Unity. Divisor.

And in like Manner may any other Divisor be found to any given Factor.

A R T. X.

To extract the Square Root by the Lines c and d.

THIS is performed by Inspection when the Lines c and d are applied even at both Ends, as I observed in Page 21 ; for when Unity at the Beginning of c is set even to Unity at the Beginning of d, you have a Table of Squares and their Roots ; for against any Square Number upon c you have the Root upon d, and, on the contrary, against any Root on d you have the Square on c : But here it will be necessary to observe the following Directions :

1. If the given Square Number consists of an odd Number of Places of Integers, seek it on the first Radius of the Line c, and against it you have the Root on d.

D 3

2. If

2. If the given Square Number consists of an even Number of Places of Integers, then seek it in the second Radius of the Line c, and against it on the Line d is the Root required.

Examples in the 1st Case.

Against $\left\{ \begin{array}{l} 9 \\ 144 \\ 207,36 \\ 90000 \end{array} \right\}$ in the 1st radius up- on the line c is $\left\{ \begin{array}{l} 3 \\ 12 \\ 14,4 \\ 300 \end{array} \right\}$ the root on the line d.

Examples in the 2d Case.

Against $\left\{ \begin{array}{l} 36 \\ 19,46 \\ 2150,42 \\ 250000 \end{array} \right\}$ in the 2d radius of the line c is $\left\{ \begin{array}{l} 6 \\ 4,4 \\ 46,37 \\ 500. \end{array} \right\}$ the root upon the line d.

A R T. XI.

To extract the Cube Root by the Lines d, and e.

THIS is also done by Inspection when the Lines d and e are set even at both Ends, viz. Unity at the Beginning of d to Unity at the Beginning of e. In this Position those Lines are in Effect a Table of Cubes and their Roots; for against any Cube on e is the Root upon d, and on the contrary, against any Root on d is the Cube upon e. The only Difficulty in this Work is to know on which of the three Radius's of the Line e you are to find your Cube Number; and in which there are three Cases.

C A S E I.

If the given Number consists of 1, 4, or 7 Places of Integers, find it on the first Radius of the Line e, and against it is the Root on d.

C A S E II.

If the given Number consists of 2, 5, or 8 places of Integers, find it on the second Radius of the Line e, and against it on the Line d is the Root required.

C A S E III.

When the given Number consists of 3, 6, or 9 Places of Integers, then find it on the third Radius of the Line e, and against it on d is the Root required.

Examples

Examples in the 1st Case.

	Cubes.		Roots.
Against	$\left\{ \begin{array}{l} 3.375 \\ 8. \\ 1728. \end{array} \right\}$	$\left\{ \begin{array}{l} \text{on the 1st ra-} \\ \text{dius of the} \\ \text{line } \pi \text{ is} \end{array} \right\}$	$\left\{ \begin{array}{l} 1.5 \\ 2. \\ 12 \end{array} \right\}$ on π .

Examples in the 2d Case.

	Cubes.		Roots.
Against	$\left\{ \begin{array}{l} 27 \\ 35.937 \\ 10648. \end{array} \right\}$	$\left\{ \begin{array}{l} \text{on the 2d ra-} \\ \text{dius of the} \\ \text{line } \pi \text{ is} \end{array} \right\}$	$\left\{ \begin{array}{l} 3. \\ 3.3 \\ 22. \end{array} \right\}$ on π .

Examples in the 3d Case.

	Cubes.		Roots.
Against	$\left\{ \begin{array}{l} 166.375 \\ 729 \\ 274625 \end{array} \right\}$	$\left\{ \begin{array}{l} \text{on the 3d ra-} \\ \text{dius of the} \\ \text{line } \pi \text{ is} \end{array} \right\}$	$\left\{ \begin{array}{l} 5.5 \\ 9. \\ 65. \end{array} \right\}$ on π .

Thus I have given Rules and Examples in all the different Cases for finding of the Cube Root; which, if understood, will be sufficient to find the Root of any Number whatsoever.

I suppose I need not acquaint my Reader that the Number of Integers in the Root are discovered by pricking off every third (beginning at the Unit's Place) towards the left Hand; and as many Points as there are in the Cube, so many Places of Integers there will be in the Root; because I presume that he has learned to extract the Cube Root by the Pen before he attempts to do it by the Rule.

A R T. XII.

Of the Use of the Line π on the Sliding Rule, in working of Questions in triplicate Proportion.

HERE I will make use of the same Examples as those which were wrought by the Pen in Pages 17 and 18, shewing the Use of the Cube Root; by which will appear the Excellency of those Lines for working such Kind of Questions.

QUESTION I.

If an iron Bullet of four Inches Diameter weigh 9 Pounds, What will an iron Bullet of 8 Inches Diameter weigh?

OPERATION.

on π .	on π .	on π .	on π .	
As 4 :	9 :	8 :	72 =	Answer.
Diam.	Wt.	Diam.	Wt.	

D 4

QUESTION

QUESTION II.

If a Fathom of Rope of 10 Inches Circumference weigh 17 lb. How much will a Fathom weigh of 8 Inches Circumference?

OPERATION.

ON E.	ON E.	ON D.	ON E.	
As 10 :	17 ::	8 :	8.704 =	Answer.
Diam.	Wt.	Diam.	Wt.	

QUESTION III.

Given the Dimensions of a conical Tun, and the Content of the same, viz. the Top Diameter = 30 Inches, Bottom Diameter = 40 Inches, Depth = 20 Inches, and the Content 68,5 Ale Gallons; To find the Dimensions of another Tun of like Form, that would contain 90 Gallons.

OPERATION.

1. For the Top Diameter.

ON E.	ON D.	ON E.	ON D.	
As 68,5 :	30 ::	90 :	32,8 =	Top Diameter.
Cont.	Diam.	Cont.	Diam.	Sought.

2. For the Bottom Diameter.

ON E.	ON D.	ON E.		
As 68,5 :	40 ::	90 :	43,8 = Bottom Diameter.	
Cont.	Diam.	Cont.	Diam.	Sought.

3. For the Depth.

ON E.	ON D.	ON E.	ON D.	
As 68,5 :	20 ::	90 :	21,9 =	Depth required.
Cont.	Depth.	Cont.	Depth.	

And thus are the Answers found to be the same as those found by the Pen; but with a vast deal more Ease and Expedition.

CHAR.

C H A P. V.

Of a C I R C L E.

Definitions of a Circle, and some of its Properties, with the Relation they bear to each other.

1. **A** Circle is the most capacious of all Figures, that is, it comprehends the most Space in the shortest Boundary Line of any Figure whatsoever.

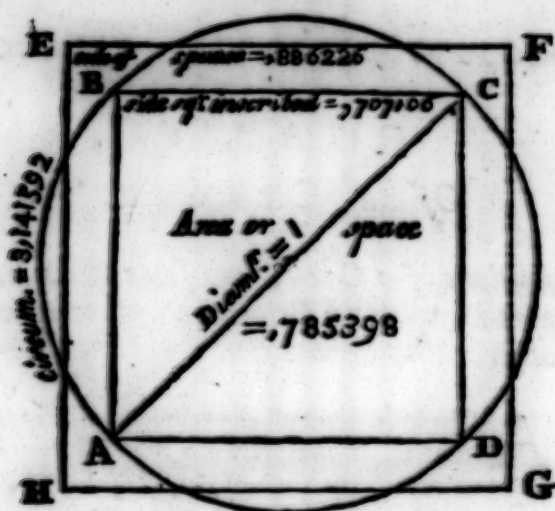
2. The Areas of all Circles are in Proportion to one another as the Squares of their Diameters.

3. If one Circle be double in Diameter to that of another Circle, the same Circle will contain four Times the Area or Space of the other.

4. The Diameter of a Circle is to its Circumference as 1 is to 3.141592, &c. that is, supposing the Diameter of a Circle to be 1, the Circumference of that Circle will be 3.141592, &c. This Proportion of the Diameter to the Circumference of a Circle was first found by one *Van Ceulen*, a *Dutchman*, who, with a great deal of Labour and Trouble, found it true to 30 Places of Decimals. Others, since him, have found it true to 100 Places of Decimals: But the first 6 Decimals being sufficient for most Purposes, therefore I have omitted the rest. And here it must be observed, that upon this Supposition are grounded all the Proportions relating to a Circle in the following Work, *viz.* If the Diameter of a Circle be 1, the Circumference of that Circle will be 3.141592.

Having premised these Things, I shall, in the next Place, shew the Method of finding the Relation that the Side of the greatest inscribed Square, the Area, and the Side of the Square equal, bears to the Diameter of a Circle.

Let the Diameter of the Circle A, B, C, D annexed $= 1 = AC$, and the Circumference $= 3.141592$, to find the Side of the Square inscribed $= AB = BC = CD = DA$, the Area or Space comprehended in the Circle, and the Side $EF = FG = GH = HE$ of the Square equal,



1. To find the Side of the greatest inscribed Square = B C.

R U L E.

It is demonstrated in 1 *Euclid*, 47, that in every right angled Triangle the Square of the Hypotenuse is equal to the Sum of the Square of the other two Sides of the Triangle. Now in the Triangle A B C, A B and B C are the two Sides of the Triangle, as well as the two Sides of the inscribed Square, and the Diameter A C = 1 is the Hypotenuse: Therefore half 1 = .5, whose Square Root is = B C, or A B required.

O P E R A T I O N.

.50(.707106 = Side of the inscribed Square.

$$\begin{array}{r} 1407 \overline{) 10000} \\ \underline{9849} \end{array}$$

$$\begin{array}{r} 14141 \overline{) 15100} \\ \underline{14141} \end{array}$$

$$\begin{array}{r} 1414206 \overline{) .9590000} \\ \underline{8485236} \end{array}$$

Rem. 1104764

2. To find the Area of a Circle whose Diameter is Unity, or the Area of any other Circle.

R U L E.

Multiply half the Circumference by half the Diameter, and the Product will be the Area required.

O P E R A T I O N.

$$\text{Half of } 3.141592 = 1.570796$$

$$\text{Half of } 1 = \underline{\quad .5 \quad}$$

$$\text{Area of a Circle} = .785398 \text{ whose Diam.} = 1.$$

3. 10

3. To find the Side of a Square equal in Area to that of a Circle whose Diameter is Unity, equal to $EF = FG$, &c.

R U L E.

If the Side of a Square be multiplied by itself the Product will be the Area of that Square; and having already found that, when the Diameter of a Circle is 1, the Area is $= .785398$: Therefore extract the Square Root of that Area, and it will be the Side of the Square equal required $= EF = FG$, &c.

O P E R A T I O N.

$.785398(.886226 =$ Side of a Square
64 equal.

168)1453

1344

1766) 10998

10596

17722). 40200

35444

177242)475600

354484

1772446)12111600

10734676

Remains .1376924

One of the Sides of a Square equal in Area to that of a Circle whose Diameter is 1, being .886226, which $\times^4$ by 4 $= 3.544904$. the Sum of the four Sides. Now the Circumference of the same Circle is but 3.141592, which is less by .403312 than the Sides of a Square equal; which proves that the Circle is the most capacious Figure.

Having found that if the Diameter of a Circle be 1,

{	The Circumference will be	3.141592
	The Side of the greatest inscribed Square	.707106
	The Side of a Square equal	.886226
	The Area	.785398

I shall now proceed to shew the Uses of these Factors in finding the like Parts of a Circle of any other Diameter both by Pen and Rule.

Suppose

Suppose the Diameter of a Circle was 30 Inches, What will the Circumference, the Side of the Square inscribed, Side of a Square equal, and Area of that Circle be?

1. For the Circumference by the Pen.

R U L E.

Multiply the Circumference of the Circle whose Diameter is Unity by the given Diameter, and the Product will be the Circumference of the Circle required.

O P E R A T I O N.

$$\begin{array}{rcl}
 \text{Circumference of a Circle whose Dia-} & \left. \begin{array}{l} \text{meter} = 1 = \text{---} \end{array} \right\} & 3.141592 \\
 \text{Given Diameter} & \text{---} & 30 \\
 \text{Circumference required} = & \text{---} & 9.4247760
 \end{array}$$

By the Rule.

Set Unity on A to 3.14 on B, and against 30 on A is 9.424 on B, the same as before.

The Rule being thus set it is a Table; for against any Diameter on A is the Circumference on B; *e contra*, against any Circumference on B is the Diameter upon A: As for Example,

$$\begin{array}{ccc}
 \text{Against these} & \left\{ \begin{array}{l} 20 \\ 24 \\ 27 \\ 33 \\ 36 \\ 40 \end{array} \right\} & \text{Are these Cir-} \\
 \text{Diameters} & & \text{cumferences} \\
 \text{upon A} & & \text{upon B} \\
 & & \left\{ \begin{array}{l} 62.83 \\ 75.39 \\ 84.82 \\ 103.67 \\ 113.09 \\ 125.66 \end{array} \right\}
 \end{array}$$

2. To find the Side of the inscribed Square.

Rule by Pen.

Multiply the Side of the inscribed Square whose Diameter is Unity, by the given Diameter, and the Product will be the Side of the inscribed Square required.

O P E R A T I O N.

$$\begin{array}{rcl}
 \text{Side of the Square whose Dia-} & \left. \begin{array}{l} \text{meter is } 1 = \text{---} \end{array} \right\} = & .707106 \\
 \text{Given Diameter} & \text{---} & 30 \\
 \text{Side of the inscribed Square required} = & \text{---} & 21.213180
 \end{array}$$

By the Rule.

Set Unity on A to .707 on B, and against 30 on A is 21.21 on B, the same as before.

The

The Rule, being thus set, is likewise a Table; for against any Diameter on A is the Side of the Square inscribed upon B; *e contra*, against any Side of an inscribed Square upon B is the Diameter on A: As for Example,

Against these Diameters upon A	$\left\{ \begin{array}{c} 20 \\ 24 \\ 27 \\ 33 \\ 36 \\ 40 \end{array} \right\}$	Are these Sides upon B	$\left\{ \begin{array}{c} 14.14 \\ 16.97 \\ 19.09 \\ 23.33 \\ 25.45 \\ 28.28 \end{array} \right\}$
--------------------------------------	--	---------------------------	--

3. To find the Side of a Square equal.

Rule by Pen.

Multiply the Side of a Square equal, whose Diameter is Unity, by the given Diameter, and the Product will be the Side of the Square equal required.

OPERATION.

Side of a Square equal whose Dia- meter is 1	_____	} = ,886226
Given Diameter	_____	
		30
Side of the Square equal required	_____	= 26.586780

By the Rule.

Set Unity on A to ,886 on B, and against 30 on A is 26.58 on B, the same as before.

The Rule being thus set, it is also a Table; for against any Diameter on A is the Side of the Square equal on B; *e contra*, against any Side of a Square equal on B is the Diameter on A: As for Example,

Against these Diameters on A	$\left\{ \begin{array}{c} 20 \\ 24 \\ 27 \\ 33 \\ 36 \\ 40 \end{array} \right\}$	Are these Sides of a Square equal on B	$\left\{ \begin{array}{c} 17.72 \\ 21.26 \\ 23.92 \\ 29.24 \\ 31.90 \\ 35.44 \end{array} \right\}$
------------------------------------	--	--	--

4. To find the Area.

Rule by the Pen.

Multiply the Area of a Circle whose Diameter is Unity by the Square of the given Diameter, and the Product will be the Area required.

O P E R A T I O N .

$$\text{Given Diameter} = \begin{array}{r} 30 \\ 30 \\ \hline \end{array}$$

$$\text{Square of Diam.} = 900$$

$$\begin{array}{rcl} \text{Factor for the Area} & = & .785398 \\ \text{Square of Diam.} & - & \underline{\hspace{1cm}} \\ & & 900 \end{array}$$

$$\text{Area required} \text{ --- } = 70,6858200$$

By the Rule.

Set Unity on D to .7854 on C, and against 30 on D is 70.68 on C, the same as before.

The Rule being thus set, it is a Table; for against any Diameter on D is the Area on C; and *contra*, against any Area on C is the Diameter on D: As for Example,

$$\begin{array}{l} \text{Against these} \\ \text{Diameters} \\ \text{on D} \end{array} \left\{ \begin{array}{l} 2 \\ 4 \\ 8 \\ 16 \\ 32 \end{array} \right\} \begin{array}{l} \text{Are those Areas} \\ \text{on C.} \end{array} \left\{ \begin{array}{l} 3.14 \\ 12.50 \\ 50.26 \\ 201.06 \\ 804.26 \end{array} \right\}$$

In the above Examples the Diameters are doubled every Time, and the Areas answering each double Diameter are fourfold the other; which is a Proof that any Circle double in Diameter to another will comprehend four Times the Area or Space of that other.

Having found a Factor for the Square of the Diameter of any Circle, *viz.* .785398, there may be a Divisor found by Chap. iv, Art. ix, that will answer the same End, and which, in many Cases, will be more commodious; for if Unity be divided by .785398 the above Factor, the Quotient will be 1.27324 the respective Divisor, and it will be the same Thing whether the Square of the Diameter is multiplied by .785398, or divided by 1.27324; for both the Product and Quotient will be equal to the Area in square Inches, or in the same Measure that the Diameter was taken in. But on account of the different Capacities of Gallons, Bushels, and Feet, it will be necessary to find Factors and Divisors for each of these different Denominations, because the Dimensions are all taken in Inches.

CH A P. VI.

Of finding Divisors and Gage-points.

A R T. I.

To find Divisors and Gage-points for Circles.

ACCORDING to Act of Parliament there is contained in a *Winchester* Gallon of Ale 282 solid Inches, in a Gallon of Wine 231 solid Inches, and in a Bushel of Malt 2250.42 solid Inches; consequently in a Gallon of Malt are contained 258.8 solid Inches, which is the $\frac{3}{4}$ Part of 2150.42; and it being found, that if the Diameter of a Circle be 1 Inch the Area thereof will be .785398, which may be called .7854 decimal Parts of an Inch: Therefore if the Capacities of each Gallon and Bushel, &c. be divided by .7854, the several Quotients will be proper Divisors for the Square of any Diameter to reduce the Area into Ale, Wine, and Malt Gallons or Bushels, &c. and the Square Roots of those Divisors will be the Gage-points for Circles.

See the Work.

Div ^r .	Div ^d .	Quot ⁿ .	S. R ⁿ .	
.7854)	282.0000	(359.05	18.95	Ale Gall.
.7854)	231.0000	(294.12	17.15	Wine Gall.
.7854)	268.8000	(342.24	18.5	Malt Gall.
.7854)	2150.4200	(737.92	52.32	Malt Bush.
.7854)	227.0000	(289.	17	Malt Tun Gallons.

The Quotients are the Divisors, and the Square Roots the Gage-points. - In like Manner may any other Divisor or Gage-point be found, when the solid Capacity of the Integer is given, whether it be a Gallon, a Bushel, a Pound Weight, or a Foot, &c. And in this Manner were all the Divisors and Gage points in the following Table found.

A R T. II.

To find Factors for Circles.

SINCE the same may be performed by Multiplication as by Division, and there may be Factors found which will answer the same End as the Divisors before found; therefore, to find the same divide .7854 by the solid Capacities of each

and Bushel, &c. and the Quotients will be proper Multipliers for the Square of the Diameter of any Circle to reduce the Area of that Circle into Ale, Wine, Malt Gallons or Bushels, &c.

See the Work.

Div ^r .	Div ^d .	Quot ⁿ .	
282.	) .7854	(.002785	Common Factors for the Square of the Diam. for
231.	) .7854	(.003389	
268.8	) .7854	(.002922	
2150.42	) .7854	(.000365	
227.	) .7854	(.00346	
			Ale Gall. Wine Gall. Malt Gall. Malt Bush. Malt Tun Gallons.

In like Manner were the other Factors for Circles found, which are inserted in the following Table.

A R T. III.

To find Factors for Squares.

THIS is done by dividing of Unity by the solid Capacity of each Gallon, Bushel, Peck, &c. See the following Examples.

282.	) 1,000000	(.003546	Common Factors for Squares	Ale Gall. Wine Gall. Malt Gall. Malt Bush. Malt Tun Gallons.
231.	) 1,000000	(.004329		
268.8	) 1,000000	(.003720		
2150.42	) 1,000000	(.000465		
227.	) 1,000000	(.0044		

And in like Manner were all the other Factors for Squares found, which are inserted in the following Table.

A R T. IV.

To find Gage-points for Squares.

THESE Gage-points are no other than the Square Roots of the solid Capacities of each Gallon, Bushel, &c. Therefore extract the Square Root from the Number of Inches in a Gallon or Bushel, &c. and it is done. As for Example,

282.	16.79
231.	15.19
268.8	16.40
2150.42	46.37
227.	15.07

ART. V.

A Table of Factors, &c.

	Multipliers, Divisors, and Gage-points for Squares and Circles.	
	Factors for	Gage-points for
	Squares.	Circles.
Each the Area of Unity	1	1.128
A Superficial Foot	1006944	13.34
A Solid Foot	100578	46.9
Ale Gallon	1003646	18.95
Wine Gallon	1003399	17.15
Malt or Corn Bushel	100465	15.19
Malt Gallon	1003710	46.37
Malt Ten Gallon	1004495	16.39
A Peck of Hard Sops	1036845	13.1
A Peck of Green Sops	1038916	3.31
A Peck of White Soft Sops	1039123	5.06
A Peck of Tallow net	1039123	5.55
A Peck of Cheese net	1039123	5.6
A Peck of Dry Starch	1039123	5.9
A Peck of Flint Glass	1039123	6.35
A Peck of White Glass	1039123	3.35
A Peck of Green Glass	1039123	3.74
		1.48
		1.94

Having found proper Factors and Divisors both for Squares and Circles, as in the above Table, the Area of any square or circular Figure may be found in any of the Denominations therein mentioned by the following general Rules:

1. For a square, or an oblong Figure.

Multiply the two Sides of any square or oblong Figure together, and their Product multiply or divide by its proper Factor or Divisor for Squares in the above Table; then the last Product or Quotient will be the Area of the same Denomination as the Factor, &c. made use of.

2. For circular Figures.

Multiply or divide the Square of the Diameter of any Circle, or the Rectangle of the transverse and conjugate Diameters of the Ellipse, by its proper Factor or Divisor in the above Table, and the Product or Quotient will be the Area of the same Denomination as the Factor, &c. made use of.

C H A P. VII.

Of finding the Areas of Superficies.

A R T. I.

*To find the Area of a Geometric Square,
like the Figure annexed.*

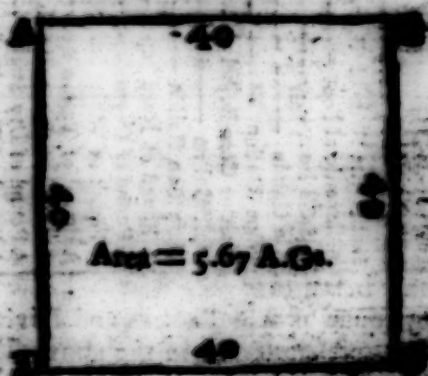
Rule by Pen.

Multiply one of the Sides of the Square by it-
self, and that Product multiply or divide by
the Factor, &c. for Squares in Page 49, and the
Product or Quotient will be equal to the Area of
the same Kind as the Factor or Divisor made use
of.

E X A M P L E.

Let the Side of the Square A B C D equal 40
Inches, what is the Area in Ale Gallons?

S Q U A R E.



O P E R A T I O N.

By Division.

By Multiplication.

$$40 = 40$$

$$40 = 40$$

$$40 \div 40 = 1$$

$$144$$

$$144$$

$$144$$

$$144$$

$$144$$

$$144$$

$$144$$

$$144$$

$$144$$

$$144$$

$$144$$

$$144$$

$$144$$

$$\text{Square Factor} = 40$$

$$\text{Square of } 40 = 1600$$

$$\text{Area} = 5.67$$

Of finding the Areas of Superficies. 31

I have wrought this Example both by the Divisor and Factor for Square Ale Gallons, and the Area comes out the same, viz. 5.67 Ale Gallons both Ways. But in my future Operations I shall only make use of the Divisor, because by that the Area is found with the fewest Figures in most Cases. In like Manner may the Area of the Square be found in any of the other Denominations mentioned in the Table by making use of the respective Factors, &c. for if, instead of 282, I had divided by 231, 2150.42, the Area would have been found in Wine Gallons; or Malt Bushels, and so of any of the rest; which is so plain, that I shall confine myself in all my Operations to one of the Denominations, which if understood, any of the others may be performed in the same Manner.

To find the same by the Rule.

The Proportion is,

ON A.	ON B.	ON A.	ON B.
As 282 :	40 ::	40 :	5.67 = before found.
Sq. Diam.	Side.	Side.	Area.

Or thus,

Set Unity on c to the Square Gage-point on a, and against any Side of a Square on d is the Area on c.

OPERATION.

ON B.	ON C.	ON D.
As 16.79 :	1 ::	40 : 5.67, as before.
Sq. Gage-pt.	Unity.	Side of Sq. Area.

The Rule being thus set, it is a Table: for against any Side of a Square upon d is the Area upon c.

ART. II.

To find the Area of the Parallelogram

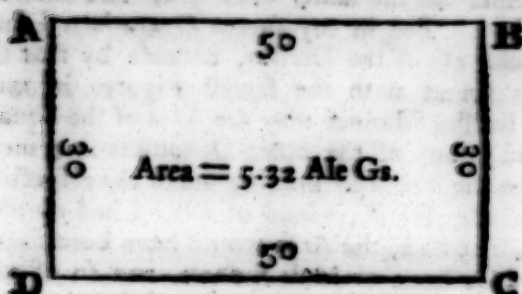
A B C D.

Rule by Pen.

Measure the length by the shortest Side, and multiply or divide by the same as is done in Page 49, and the Product will be the Area required.

52 Of finding the Areas of Superficies.

PARALLELOGRAM.



EXAMPLE.

Let the longest Side of the Parallelogram = 50 Inches = AB , the shortest Side = 30 Inches = BC , what is the Area of it in Ale Gallons?

OPERATION.

Longest Side $AB = 50$

Shortest Side $BC = 30$

Sq. Divisor = 282)1500)5.32 fere the Area in Ale Gallons.

$$\begin{array}{r}
 1410 \\
 \hline
 .900 \\
 846 \\
 \hline
 .540 \\
 564 \\
 \hline
 \end{array}$$

Remains — 24

The Area above found is not quite 5.32: But it is nearer to 5.32 than to 5.31, and because in the Practice of Gaging we use but two decimal Places of Figures, therefore we take the nearest to the second Place, whether it be more or less.

The Proportion by the Rule.

ON A.	ON B.	ON A.	ON B.
As 282	: 50	:: 30	: 5.32 the same as
Sq. Divi-	Long	Short	Area before.
for.	Side.	Side.	

A R T. III.

To find the Area of a Rhombus whose Sides are equal and parallel, and two of the opposite Angles are acute and the other two are obtuse. See the Figure following.

RULE.

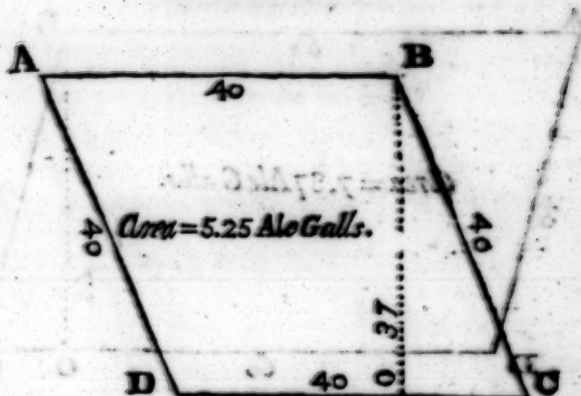
Multiply the perpendicular or shortest Distance between the two opposite Sides by one of the Sides.

Of finding the Areas of Squares. 53
the Sides, and that Product multiply or divide by
the Factors, &c. for Squares in Page 49. and the
Product or Quotient will be the Area required.

EXAMPLE.

Let the Side of the Rhombus $ABCD = 40$
Inches $= AB = BC$, and the Perpendicular AO
 $= 37$ Inches, what is its Area in Ale Gallons?

RHOMBUS.



OPERATION.

Perpendicular $AO = 37$
Side $AB = 40$
Square Div. for Ale Ga. $= 282 \overline{) 1480} 5.25 = \text{Area}$
 $\begin{array}{r} 1410 \\ \cdot 700 \\ \cdot 564 \\ \hline 1360 \\ 1410 \\ \hline \text{Remains } \cdot 50 \end{array}$

The Proportion by the Rule.

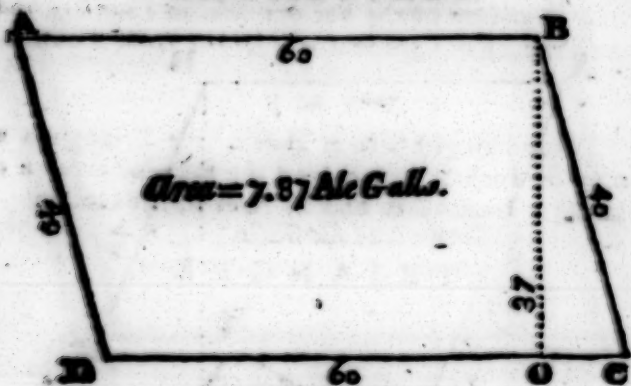
ON A. ON B. ON A. ON B.
As 282 : 40 :: 37 : 5.25 as before.
Sq. Div. Side Perpend. Area.

34 Of finding the Areas of Superficies.

A R T. IV.

To find the Area of a Rhomboides, whose two longest Sides are equal and parallel, and the two shortest Sides are also equal and parallel, two of its opposite Angles are obtuse, and the other two acute. See the following Figure.

RHOMBOIDES.



E X A M P L E.

LET the last Figure represent the Rhomboides, whose longest Sides AB and $DC = 60$ Inches, Perpendicular $BO = 37$ Inches; To find the Area in Ale Gallons.

Operation by Pen.

Perpendicular $BO = 37$

Longest Side $AB = DC = 60$

Square Ale Divisor $= 282$) 2220 (7.87 Ale Area.

1974

2460

2256

2040

1974

Remains $— .66$

The Proportion by the Rule.

ON A. ON B. ON A. ON B.

As 282 : 60 :: 37 : 7.87 = as before.

Sq. Div. Long Side. Perp. Area.

A R T. V.

To find the Area of a plain Triangle.

D E F I N I T I O N.

A Triangle consists of three Sides and three Angles: of which there are two Sorts, the Equilateral and the Scalene.

Of finding the Areas of Superficies.

a rectangled Triangle, and the other an oblique angled Triangle: But as one general Rule will serve for both Sorts, I shall only give an Example of an oblique angled Triangle, which must be divided into two rectangled Triangles, in order to find the Area thereof.

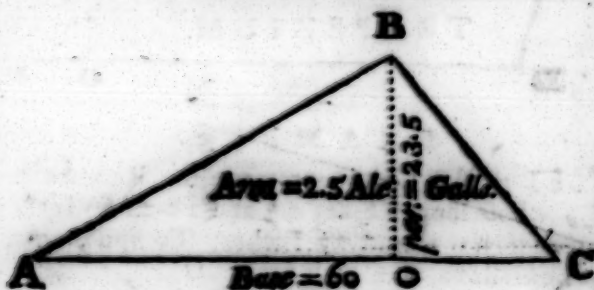
R U L E.

From the greatest Angle let fall a Perpendicular on the longest Side, and multiply half that Perpendicular by the longest Side, or half the longest Side by the whole Perpendicular, and that Product multiply or divide by the Factors, &c. in Page 49. and the Product or Quotient will be the Area required.

E X A M P L E.

Let the longest Side or Base of the Triangle $A B C = 60$ Inches $= A C$, and the Perpendicular $B O = 23.5$ Inches; to find its Area in Ale Gallons.

T R I A N G L E.



O P E R A T I O N.

Perpendicular $— B O = 23.5$

$\frac{1}{2}$ Base $— A C = 30$

Square Ale Divisor $= 282 \overline{) 705.0} \{ 2.5 = \text{Area in Ale Ga.}$

564

1410

1410

0

The Proportion by the Rule.

ON A.	ON B.	ON A.	ON B.
As 282	: 30	:: 23.5	: 2.5 = as before.
Sq. Div.	$\frac{1}{2}$ Base.	Perp.	Area.

A R T. VI.

To find the Area of a Trapezium.

D E F I N I T I O N.

A Trapezium is composed of four Sides and four Angles: Such the Sides and the Angles are supposed. See the following Figure.

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R U L E.

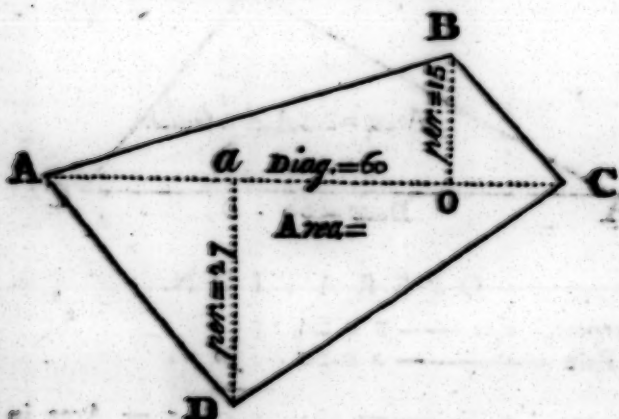
Divide the Trapezium into two Triangles by a right Line drawn from one Angle to its opposite Angle, which is called a Diagonal, and divides the Trapezium into two Triangles, then let fall a Perpendicular from each of the other Angles upon that Diagonal, which is a common Base to both Triangles.

2. Multiply half the Diagonal by the Perpendiculars, or half the Sum of the Perpendiculars by the whole Diagonal, and that Product multiply or divide by the Factors, &c. in Page 49, and the Product or Quotient will be the Area required.

E X A M P L E.

Let the Figure $A B C D$ represent the Trapezium, whose Diagonal $A C = 60$ Inches, the Perpendicular $B O = 15$ Inches, and the Perpendicular $D a = 27$ Inches; to find the Area in Ale Gallons.

T R A P E Z I U M.



O P E R A T I O N.

Perpendicular — $B O = 15$

Perpendicular — $D a = 27$

Sum = 42

$\frac{1}{2}$ Diagonal $A C = 30$

Square Ale Divisor = 282) 1260 (4.47 = Area in Ale Ga.

1128

1320

1128

1920

1974

Remains — .54

The Proportion by the Rule.

on A. on B. on A. on B.
As 282 : 42 :: 30 : 4.47 = as before
Sq. Div. Perpend. $\frac{1}{2}$ Diag. Area.

Note

Note here, that the Areas of the five last Figures may be found by the Lines c and d on the Rule, the same as if they were Squares, if there be first a mean geometrical Proportional found between the Perpendiculars and their Bases: But the Method of finding this mean Proportional shall be the Business of the next Article.

A R T. VII.

To find a mean geometrical Proportional between two given Numbers.

R U L E.

Multiply the two given Numbers together, and extract the Square Root from their Product, which will be the Mean required.

E X A M P L E.

Let the given Numbers be 72 and 50; to find a mean Proportional between them.

O P E R A T I O N.

$$\begin{array}{r}
 72 \\
 \times 50 \\
 \hline
 3600
 \end{array}
 \begin{array}{l}
 60 = \text{Mean required.} \\
 36 \\
 \hline
 000
 \end{array}$$

By the Rule.

Set one of the given Numbers upon c to the same Number upon d, then against the other given Number upon c is the Number sought upon d.

Thus,

on c.	on d.	on c.	on d.
As 50 :	50 ::	72 :	60 = the same as before.
Given	Given	Given	Mean.
Num.	Num.	Num.	

Another Example by the Rule.

Find a Mean between 42 and 30.

O P E R A T I O N.

on c.	on d.	on c.	on d.
As 42 :	42 ::	30 :	35.5 = Mean required.
Given	Given	Given	Mean.
Num.	Num.	Num.	

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This Mean last found is a Mean proportional between the Sum of the Perpendiculars and half of the Diagonal of the Trapezium, by which the Area of that Figure may be found by the Lines *c* and *a*, according to the following Proportion:

ON D. ON C. ON D. ON C.
As 16.79 : 1 :: 35.5 : 4.47 = Area found by
Sq. Gage- Unity. Mean. Area. the Lines *a*
point. and *b*.

And in like Manner may the Area of the Parallelogram, Rhombus, Rhomboides, and Triangle be found.

A R T. VIII.

To find the Area of any regular Polygon.

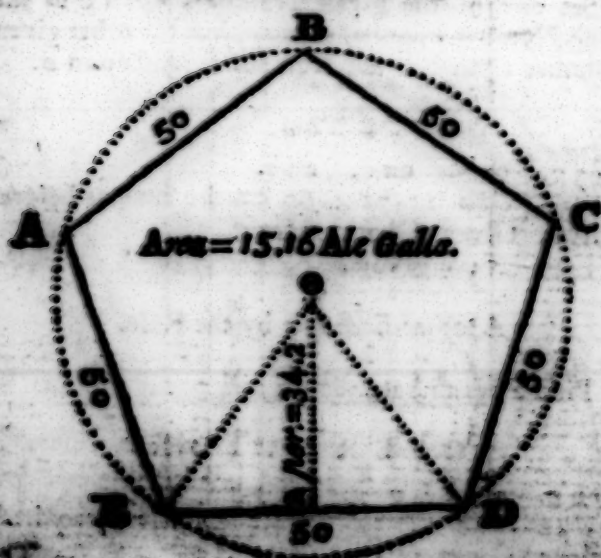
DEFINITION.

ALL Figures that have more than four Sides, and those Sides equal, and so situated that a Circle will pass thro' all its Angles, are called Regular Polygons, and are named according to the Number of Sides they consist of, viz. if it has five Sides it is called a Pentagon, if six Sides a Hexagon, if seven Sides a Heptagon, &c. as may be seen by the following Table of Polygons.

R U L E.

From the Center of the Polygon let fall a Perpendicular upon one of its Sides, which will be the Perpendicular of a Triangle, and the Side will be the Base; then find the Area of that Triangle as was shewn in Art. v. of this Chapter; and because there are as many Triangles as there are Sides in the Polygon, multiply the Area of the Triangle by the Number of Sides, and the Product will be the Area of the Polygon required.

P E N T A G O N.



EXAM.

Of finding the Area of Superficies: 199

EXAMPLE.

Let the Figure annex'd represent a Pentagon, whose Sides $AB = BC = CD = DE = EA = 50$ Inches; and the Perpendicular $ea = 34.3$ Inches, what is its Area in Ale Gallons?

OPERATION.

Perpendicular $ea = 34.3$

$\frac{1}{2}$ of one of the Sides $= 25$

1710

684

Sq. Ale Div. $= 282$ 855,0 (3,032 = Area of

846

5

the Tr.

E & D.

.900

15.160

= Area of

846

the Pen-

tagon in

Ale Ga.

540

564

Remains $= 24$

But the Areas of any of the Regular Polygons mentioned in the following Table may be found without knowing the Perpendiculars at all; as will be shown hereafter.

The Table being made from a trigonometrical Calculation, therefore I have omitted to shew the Method of its Construction; for it would be of no Use to a Person who does not understand the Rules of Plain Trigonometry; and such as do may easily find it out.

A Table of Regular Polygons.

No. of Sides.	Form of the Poly- gon.	Area, Sq. Inches.	Area, Ale Ga.	Area, Wine Cr.	Area, Malt Bu.
3	Triangle	11.72	.00469	.00744	.00975
4	Quadrangle	2.198	.00978	.01488	.01950
5	Pentagon	3.631	.01583	.02577	.03364
6	Hexagon	4.558	.02170	.03569	.04671
7	Heptagon	6.183	.02985	.04976	.06479
8	Octagon	7.687	.03787	.06311	.08377
9	Nonagon	9.361	.04785	.08085	.01068
10	Decagon	11.196	.05970	.09847	.01320

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The Use of the preceding Table.

Multiply the Square of the Side of any Regular Polygon mentioned in the Table by the common Factor belonging to that Polygon, and the Product will be the Area in Inches, Ale Gallons, Wine Gallons, or Malt Bushels respectively.

For Example, Let the Side of a Pentagon be 50 Inches, what is its Area in Ale Gallons?

OPERATION.

Side of the Pentagon = 50

50

Square of the Side = 2500

The Tabular Number for Ale Gs. = .006099

Square of the Side ——— = 2500

3049500

12198

The Area in Ale Gs. = 15.247500

Which is nearly the same as that found by letting fall the Perpendicular.

In like Manner may the Area of any of the other Regular Polygons mentioned in the Table be found in any of the Denominations there mentioned; and if the Area were required in any other Denomination not there specified, it is only dividing the Area of that Polygon whose Side is an Inch square by the square Inches contained in a Foot, Pound or Gallon, &c. and you will have a common Factor for that Denomination.

All other right-lined Figures consisting of more than four equal Sides are called Irregular Polygons. To find the Area of such Figures, draw a diagonal Line from one Angle to the opposite Angle, and find the Area of each Triangle by the foregoing Rules and the Area of each Triangle, which it is divided into, and the Sum of the Areas of the several Parts will be equal to the Area of the whole Figure.

ART. IX.

To find the Area of a Circle.

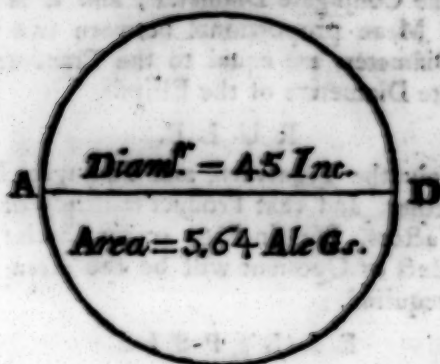
RULE.

SQUARE the Diameter of the Circle, and that Square multiply or divide by the Factors, &c. in Page 49 for Circles, and the Product, or Quotient will be the Area required.

CIRCLE

Of finding the Areas of Superficies. 61

CIRCLE.



EXAMPLE.

Let the Diameter AD of the above Circle = 45 Inches, what is the Area of it in Ale Gallons?

OPERATION.

Diameter $AD = 45$

Diameter $AD = 45$

225
180

Circ. Divisor = 359.05 $2025.00 \div 359.05 = 5.64$ fere = Area in Ale Ga.

1795.25

229750

215430

143200

143620

Remains = 430

The Proportion by the Rule.

~~As 1 is to 5.64 so is the Diameter of any Circle to its Area in Ale Gallons.~~
~~As 1 is to 5.64 so is the Diameter of any Circle to its Area in Ale Gallons.~~
 Only. Diameter. Area.

The Rule being thus set it is like a Table; for against any Diameter on d is the Area in Ale Gallons on c . So if Unity on c was set to any other circular Gage-point, then against any Diameter on d you will find the Area on c of the same Denomination as the Gage-point that Unity was set to.

ART. X.

To find the Area of an Ellipse or Oval.

DEFINITION.

An Ellipse is a plane figure bounded by a curved line, the area of which is to be found.

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The longest Diameter AB is called the Transverse Diameter, and the shortest Diameter CD is called the Conjugate Diameter; and it is a geometrical Mean proportional between two Circles whose Diameters are equal to the Transverse and Conjugate Diameters of the Ellipsis.

R U L E.

Multiply the Transverse and Conjugate Diameters together, and that Product multiply or divide by the Factors, &c. in Page 49 for Circles; then that Product or Quotient will be the Area of the Ellipsis required.

ELLIPSIS.



EXAMPLE.

Let the Transverse Diameter AB of the above Ellipsis = 72 inches, and the Conjugate Diameter CD = 50 inches, what is the Area of it in Ale Gallons?

OPERATION.

Transv. Diameter AB = 72

Conj. Diameter CD = 50

359.05 (3600.00 (10.03 fere = Area in Ale

35905

95000

107715

Remains 12715

The Proportion by the Rule.

ON A.	ON B.	ON A.	ON B.
As 359	: 72	:: 50	: 10.03 = as above.
Circular	Transv.	Conj.	Area.
Div.	Diam.	Diam.	

Or thus,

Find a Mean between the Transverse and Conjugate Diameter, which, in this Case, is = 60; then the Proportion will be the same as that for a Circle.

ON A.	ON B.	ON A.	ON B.
As 113.1	: 1	:: 60	: 10.03 = as before.
Gr. Circ.	Unity.	Mean.	Area.

CHAP.

CH A P. VIII.

Of finding the Contents of Solids.

AS Superficies are comprehended under Length and Breadth, so Solids are comprehended under Length, Breadth, and Depth; and tho' it be improper, in a geometrical Sense, to ascribe Thickness to a Superficies, yet Gagers always consider them as one Inch deep, and accordingly compute the superficial Content or Area in Ale Gallons, &c. in solid Measure.

Now by having the Area or Content given at one Inch deep, it will be an easy Matter to find the Content of any solid Body at any Depth, provided the Bases of that Body are equal, and straight Lines may be every where applied from one Base to the other; for if the Area or Content at one Inch deep be multiplied by the whole Depth, the Product will be the solid Content of that Body.

From hence it will be very easy to find the Solidity of any Body that hath any of the Figures in the last Chapter for its Bases, which Figures I shall here make use of for the Bases, supposing them to have Depth, as well as Length and Breadth.

A R T. I.

To find the Content of a Solid whose Bases are either Squares or Parallelograms, both equal and parallel to each other, and of equal Substances thro' every Part of the Depth.

R U L E.

Multiply the Length of the Base by the Breadth, and that Product by the Depth, and divide that Product multiply or divide by the Factors, &c. for Squares in Page 49, and the Product or Quotient will be the Content required, of the same Kind as the Factor, &c. made use of.

Or,

Find the Area of the Base by Art. i. and ii. of the last Chapter, and multiply that Area by the Depth, and the Product will be the Content.

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Example I. for a Square Base.

Let the Figure in Art. i. of the last Chapter represent the Base of the Solid whose Side is = 40 Inches, and suppose its Depth = 10 Inches, what will its Content be in Ale Gallons?

OPERATION.

$$\begin{array}{r}
 40 \\
 40 \\
 \hline
 \text{Square of the Side} = 1600 \\
 \text{Depth} \quad \quad \quad = 10 \\
 \hline
 \text{Square Divisor} = 282 \overline{)16000} 56.7 = \text{Content in Ale Gs.} \\
 \underline{1410} \\
 1900 \\
 \underline{1692} \\
 2080 \\
 \underline{1974} \\
 \text{Remains } 106
 \end{array}$$

The Area of the Base of this Solid was found in Art. i. of the last Chapter to be = 5.67, which multiplied by 10, the Depth, = 56.7 = the Content in Ale Gallons, the same as above found.

The Proportion by the Rule.

$$\begin{array}{ccccccc}
 \text{on D.} & \text{on C.} & \text{on C.} & \text{on D.} & & & \\
 \text{As } 10.79 : 10 :: 40 : 56.7 = \text{as before.} \\
 \text{Sq. Gage-pt. Depth. Side of Content.} \\
 \text{Square.}
 \end{array}$$

Example II. for an Oblong Base.

Let the Figure in Art. ii. of the last Chapter represent the Base of the Oblong Solid, whose longest Side A B = 50 Inches, shortest Side A D = 30 Inches, and suppose the Depth = 10 Inches, what is its Content in Ale Gallons?

OPERATION.

$$\begin{array}{r}
 \text{Side A B} \quad \quad \quad = 50 \\
 \text{Side A D} \quad \quad \quad = 30 \\
 \hline
 1500 \\
 \text{Depth} \quad \quad \quad = 10 \\
 \hline
 \text{Square Divisor} = 282 \overline{)15000} 53.2 = \text{Content in Ale Gs.} \\
 \underline{1410} \\
 900 \\
 \underline{846} \\
 540 \\
 \underline{522} \\
 18
 \end{array}$$

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The Area of the Base was found in Art. ii. of the last Chapter to be $= 5.32$; which multiplied by 10 $= 53.2$, the Content in Ale Gallons, the same as that before found.

The Proportion by the Rule is the same as that for a Square Base, when there is first a Mean Proportional found between the longest and shortest Sides of the Base: As for Example,

1. For the Mean Proportional.

on C. on D. on C. on D.
As 50 : 50 :: 30 : 38.73 = Mean sought.
L. Side. L. Side. S. Side. Mean.

2. For the Content.

on D. on C. on D. on C.
As 16.79 : 10 :: 38.73 : 53.2 = as by Pen.
S. Gage-pt. Dth. Mean. Content.

Or the Content may be found by the Lines A and B, by first finding the Area of the Base, as in Art. ii. of the last Chapter, and then multiplying that Area by the Depth; which Product will be the Content. This is so plain, that it requires no Examples.

In like Manner may the Content of any other right lined Solid be found, (whether it has a Triangle, a Rhombus, or Rhomboides; or if it has any of the Polygons, either regular or irregular, for its Bases) if by the foregoing Rules you find the Area of its Base, and multiply that Area by its Depth.

A R T. II.

To find the Content of a Solid that has two equal Circles for its Bases, commonly called a Cylinder, or if it has two equal Ellipses for its Bases called a Cylindroid, by having the Depth and Diameters given. It is supposed that the Bases are parallel, and that the Bodies are of an equal Diameter throughout the whole Depth.

R U L E.

By Art. iii. and x. of the last Chapter find the Area of the Base, which Area multiply by the Depth, and the Product will be the Content of the Solid required.

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Example I. for the Cylinder.

Let the Figure in Page 61 represent the Bases of the Cylinder, whose Diameter is = 45 Inches = A D, and suppose the Depth = 12 Inches: To find the Content in Ale Gallons.

OPERATION.

$$\begin{array}{r}
 \text{The Area of the Base, as found by Art. } \} = 5.64 \\
 \text{ix. Chap. vii. } \text{-----} \text{-----} \text{-----} \\
 \text{Depth } \text{-----} \text{-----} \text{-----} \text{-----} = 12 \\
 \hline
 1128 \\
 564 \\
 \hline
 \end{array}$$

Content in Ale Gallons = 67.68

The Proportion by the Rule.

$$\begin{array}{ccccccc}
 & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\
 \text{As } 18.95 & : & 12 & :: & 45 & : & 67.68 & = \text{ as before.} \\
 \text{C. Gage} & \text{Depth.} & & \text{Diam.} & & \text{Content.} & & \\
 \text{point.} & & & & & & &
 \end{array}$$

Example II. for a Cylindroid.

Let the Figure in Page 62 represent the Bases of the Cylindroid, whose Transverse Diameter A B = 72 Inches, and Conjugate Diameter C D = 50 Inches, and suppose the Depth to be = 12 Inches, how many Gallons will it contain?

OPERATION.

$$\begin{array}{r}
 \text{The Area of the Base as found in Art. } \} = 10.03 \\
 \text{x. Chap. vii. } \text{-----} \text{-----} \text{-----} \\
 \text{Depth } \text{-----} \text{-----} \text{-----} \text{-----} = 12 \\
 \hline
 2006 \\
 1003 \\
 \hline
 \end{array}$$

The Content in Ale Gallons = 120.36

The Proportion by the Rule is the same as for a Cylinder, when first there is a Mean Proportional found between the Transverse and Conjugate Diameters.

1. For the Mean Proportional.

$$\begin{array}{ccccccc}
 & \text{on C.} & & \text{on D.} & & \text{on C.} & & \text{on D.} \\
 \text{As } 72 & : & 72 & :: & 50 & : & 60 & = \text{ the Mean Diameter.} \\
 \text{Transf.} & \text{Transf.} & & \text{Conj.} & & \text{Mean.} & & \\
 \text{Diam.} & \text{Diam.} & & \text{Diam.} & & & &
 \end{array}$$

2. For

Of finding the Contents of Solids. 67

2. For the Content.

on d. on c. on d. on c.
As 18.95 : 12 :: 60 : 120.36 = as before.
Cir. Gage- Depth. Mean. Content.
point.

Or the Content may be found by the Lines A and B at two Operations, viz. first find the Area, and then multiply that Area into the Depth, and the Product will be the Content. As for Example,

1. For the Area.

on A. on B. on A. on B.
As 359 : 72 :: 50 : 10.03 = Area;
Cir. Div. Tr. Diam. C. Diam. Area.

2. For the Content.

on B. on A. on B. on A.
As 1 : 10.03 :: 12 : 120.36 = Content before
Unity. Area. Depth. Content. found.

In like Manner may the Content be found in any other Denomination by making use of the respective Factor, &c. for that Denomination.

But the Area or Content of any square or circular Figure may be found at once setting the Rule for each, in all the different Denominations, by taking out the Slide on which the Line c is and putting it in again the reverse or backward Way, and counting the Numbers in that Order: for then if Unity on c be set to the Side of a Square, or Diameter or Mean Side of a Parallelogram, or Mean Diameter of an Ellipsis, against any Gage-point on d is the Area on c. Or if the Depth on c be set to the Side of a Square or Diameter, &c. on d, then against any Gage-point on d is the Content on c. As for Example, Let the Side of a Square or Diameter = 30 Inches, set Unity on c to 30 on d.

		Areas.		Areas.	
then against the sq. gage-pt. on d for	Alc Ga.	are on c	3.18	and against the cir. gage-pt. on d are	2.51
	Wine Ga.		3.90		3.06
	Malt Bu.		0.42		0.33
				on c.	

Suppose the Depth or 8 Inches, then set 8 on c to 30 on d.

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then against the sq. gage-pts. on D for	{ Ale gs. Wine gs. Malt bs. }	are on C	{ 25.44 31.20 3.36 }	and against the cir. gage-pts. on D are	{ 20.08 24.48 2.64 }	on C.

A R T. III.

To find the Content of a Pyramid or Cone.

D E F I N I T I O N.

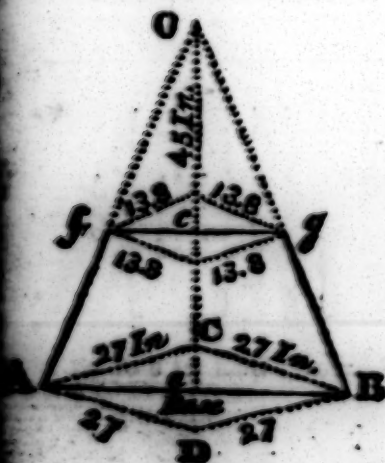
A Pyramid is a Solid whose Base is a Polygon, and its Sides are Triangles, whose Tops all meet in a Point called its Vertex.

A Cone hath a circular Base, and tapereth away to a Point called its Vertex, in such a Manner that all Lines along the flant Surface are strait Lines; the Pyramid and Cone being so nearly related to each other, that one general Rule may suffice for finding the solid Contents of both, for either of them are equal to one third Part of a Solid of the same Height, and of the same Area at the Base.

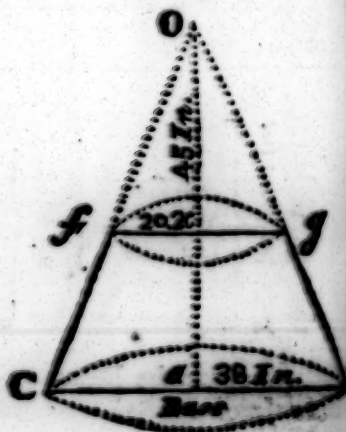
R U L E.

Find the Area of the Base by the foregoing Rules, whether it be a Circle, Square, or any of the Regular Polygons, which Area multiply by one third Part of its Height; or, which is the same, multiply the whole Height by one Third of the Area of its Base, and the Product will be the Content required.

A SQUARE PYRAMID.



A CONE.



Example

Of finding the Contents of Solids. 69

Example I. for the Square Pyramid.

Admit the Side of the Base of the Pyramid $A O B = 27$ Inches $= A C = C B = B D = D A$, and the Altitude $O a = 45$ Inches, what is its Content in Ale Gallons?

OPERATION.

Side of the Base $= A C = 27$

Side of the Base $= C B = 27$

189

54

Square Ale Divif. $= 282)729($

564

1650

1410

2400

2256

.1440

1410

Remains — .30

2.585 = Area.

15 = $\frac{1}{2}$ of $O a$.

12925

2585

38.775 = Con. in

Ale Ga.

To find the same by the Rule.

PROPORTION.

on D.	on C.	on D.	on C.	
As 16.79	: 15	:: 27	: 38.77	as above.
Sq. Gage-	$\frac{1}{2}$ Altitude.	Side of	Content.	
point.		Base.		

Example II. for the Cone.

Admit the Diameter of the Base of the Cone $c o d$ to be 38 Inches $= c d$, and its Altitude 45 Inches $= O a$, what is its Content in Ale Gallons?

OPERATION.

The Diameter $c d = 38$

The Diameter $c d = 38$

304

114

Circ. Div. $= 359.05)1444.00($

143620

.78000

71810

.61900

71810

Remains — .9910

F 3

4.022 = Area.

15 = $\frac{1}{2}$ of $O a$.

20110

4022

60,330 = Cont.

in Ale Ga.

To

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To find the same by the Rule:

The Proportion,

ON D. ON C. ON D. ON C.

As 18.95 : 15 :: 38 : 60.3 = nearly as above.

C. Gage- $\frac{1}{2}$ Altitude. Diam. Content.
point. tude.

A R T. IV.

To find the Content of the Frustrum of a Pyramid or Cone.

D E F I N I T I O N.

A Frustrum of a Pyramid, or a Cone, is the lower Part or Remainder of a Pyramid, or Cone, when the small Ends are cut off, as $Afgb$ is a Frustrum of the Square Pyramid, and $cfgd$ is a Frustrum of the Cone: See the Figures in the last Article. And as a Pyramid and Cone are nearly related to each other, so are also their Frustrums. Therefore one general Rule will serve for finding the Contents of both.

R U L E.

To three Times the Rectangle of the greatest and least Sides of the Bases of the Pyramid's Frustrum, or the greatest and least Diameters of the Cone's Frustrum, add the Square of the Difference of the Sides of the Pyramid, or the Difference of Diameters of the Cone's Frustrum Bases, and this Sum multiply by one Third of the Height, and multiply or divide that Product by the Factors, &c. for Squares or Circles in Page 49; then the Product or Quotient will give the Content required.

Example I. for a Frustrum of a Square Pyramid.

Let the lower Part of the Pyramid $Afgb$ represent the Frustrum, whose Side of the greater Base $ac = 57$ Inches, and the Side of the lesser Base $fg = 13.8$ Inches, and Altitude or Depth $ec = 21$ Inches: To find its Content in Ale Gallons.

OPERATION.

Side of the lesser Base $fx = 13.8$	27.
Side of the greater Base $ac = 27.$	13.8
	966
	276
Rectang. of the Sides $= 372.6$	264
	3
	396
	132
3 Times the Rectang. of the Sides $= 1117.8$	
Sq. of the Diff. of Sides $= 174.24$	174.24 = Sq. of Diff.
	1292.04
$\frac{1}{3}$ of the Altitude $ac = 7$	
Product $= 9044.28$	
Square Divif. $= 282)9044.28(32.07$	Cont. in Ale Ga.
	846
	.584
	564
	.2028
	1974
Remains $= .54$	

To find the same by the Rule.

By Art. vii. Chap. vii. find a Mean Proportion between the Sides of the greater and lesser Bases; then by Art. i. Chap. vii. find the Areas of those three Squares; the Sum of which multiplied by $\frac{1}{3}$ of the Depth is equal to the Content of the Frustum.

OPERATION.

Set Unity on c to 16.79 on D, then against the

	on D.	on C.	
{ Side of the greater Base $= 27$	} those	{ 2.58	
{ Side of the lesser Base $= 13.8$			{ 0.676
{ Side of the mean Base $= 19.3$			
Sum of the Areas $= 4.56$			
$\frac{1}{3}$ of the Depth $ac = 7$			
Content in Ale Gallons $= 32.032$			

The Content found by the Rule agrees very well with that found by the Pen. And as this Method will serve for either Pen or Rule, it may be used for the Pen instead of the former Rule.

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Example II. for the Frustrum of a Cone.

Let the lower Part of the Cone $c f g d$ represent the Frustrum whose Diameter at the greater Base $c d = 38$ Inches, and the Diameter at the lesser Base $f g = 20.2$ Inches, and the Altitude or Depth $a c = 21$ Inches: To find its Content in Ale Gallons.

OPERATION.

The lesser Diam. $f g = 20.2$	$c d = 38.$	
Greater Diam. $c d = 38.$	$f g = 20.2$	
1616	Diff. = 17.8	
606	17.8	
767.6	1424	
3	1246	
	178	
3 Times Rect. of } the Diams. — }	= 2302.8	
Square of Diff. —	= 316.84	Sq. Diff. 316.84
Sum — — —	= 2619.64	
$\frac{1}{3}$ of the Depth —	= 7	
Cir. Divif. = 359.05	18337.48	(51.07 = Cont. in Ale Ga.
	179525	
	.38498	
	35905	
	.259300	
	251335	
Remains —	7965	

To find the same by the Rule. First find a Mean Proportion between the greater and lesser Diameter, and then the Work will stand thus.

OPERATION.

Set Unity on c to 18.95 on d , then against the

	on d .		on c .
{	Diam. of the greater Base = 38	}	are { 4.02
	Diam. of the lesser Base = 20.2		those { 1.12
	Mean Diam. — — — = 27.7		Ans. { 2.14
Sum of the Areas — —			= 7.30
$\frac{1}{3}$ of the Depth $a c$ —			= .7
The Content in Ale Gallons — —			= 51.10

This also agrees very well with the Content before found, which Method will serve for both Pen and Rule.

The

The Answer would have been the same if I had not found a Mean Diameter: But instead thereof found a Mean Area between the Areas of the greater and lesser Diameters, and added the three Areas together as above, and multiplied their Sum by $\frac{1}{3}$ of the Depth, the Product would give the Content the same as that before found. This last Method is universal for all Frustrums of Pyramids or Cones, let their Bases be of what Form they will, whether they be Parallelograms or Ellipses, provided their Bases are parallel, and there be the same Proportion between the longest and shortest Sides, or Diameters of the greater Base, as there is between the longest and shortest Sides, or Diameters of the lesser Bases, and where strait Lines may be applied from Base to Base throughout every Part of the Depth.

A R T. V.

To find the Content of a Solid whose parallel Bases are unequal, (the one Base may be a Square the other a Parallelogram, or the one an Ellipsis and the other a Circle) the Sides being strait from Top to Bottom, whether the Bases are alike or not alike, proportional or disproportional, alike situated or inverted.

General Rule..

1. **T**O the Length, or Transverse Diameter of the greater Base add half the Length, or half the Transverse Diameter of the lesser Base, and this Sum multiply by the Breadth or Conjugate Diameter of the greater Base; the Product reserve.

2. To the Length, or Transverse Diameter of the lesser Base add half the Length, or half the Transverse Diameter of the greater Base, and this Sum multiply by the Breadth, or Conjugate Diameter of the lesser Base, and add the Product to the former reserved Product, which Sum multiply by one Third of the Depth, and multiply or divide the last Product by the proper Factors, &c. in Page 49 for Squares or Circles, and the Product or Quotient will be the Content required, of the same Kind as the Factor, &c. made use of.

E X A M P L E.

Suppose a Solid whose Bases were Parallelograms or Ellipses, and the longest Side or Transverse Diameter

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meter of the greater Base = 75 Inches, and the shortest Side or Conjugate Diameter = 50 Inches; the longest Side or Transverse Diameter of the lesser Base = 45 Inches, and the shortest Side or Conjugate Diameter = 35 Inches, and its Depth = 30 Inches: To find the Content in Ale Gallons.

1. For the Square, &c. Bases.

OPERATION.

Longest side, &c. of greater base	} = 75	Shortest side of greater base	} = 45
$\frac{1}{2}$ longest side of the lesser base		$\frac{1}{2}$ longest side of greater base	
	} = 22.5		} = 37.5
Sum	— = 97.5		82.5
Breadth of the greater base	} = 50	Breadth of the lesser base	} = 35
1 product	— = 4875.0		
2 product	— = 2887.5		4125
			2475
Sum	— = 7792.5		
$\frac{1}{3}$ of depth	= 10		2887.5
Sq. div.	= 282)77625.0(275.3 = Cont. in Ale 564 Ga.		
	2122		
	1974		
	.1485		
	1410		
	.750		
	846		
Remains	— .96		

2. For the Oval or Elliptical Bases.

OPERATION.

Cir. Divisor = 359.05)77625.00(216.2 = Cont.
71810 in Ale Ga.

	.58150
	35905
	222450
	215430
	.70300
	71810
Remains	— .1610

The

Of finding the Contents of Solids 75

The Operation is the same both for Square and Oval Bases, till you come to make use of the Factors, &c. in 49. as appears by the above work; where I find that if the Bases are Parallelograms the Content will be 275.3 Ale Gallons; but if Ellipses 216.2 Ale Gallons.

A R T. VI.

To find the Content of a Sphers or Globe.

D E F I N I T I O N.

A Sphere or Globe is two Third Parts of a Cylinder, whose Diameter and Height is equal to the Diameter of the Sphere: Therefore if the Cube of the Diameter of a Sphere be multiplied by two Third Parts of the Circular Factors, &c. in Page 49, or if it be divided by one, and half of the Circular Divisors, then the Product, or Quotient, will be equal to the Content of that Sphere. From hence it will be easy to find Factors and Divisors for the Cube of the Diameter of any Sphere for all the different Denominations in Page 49. As for Example,

1st, For Factors.

Facts. for Cylind.

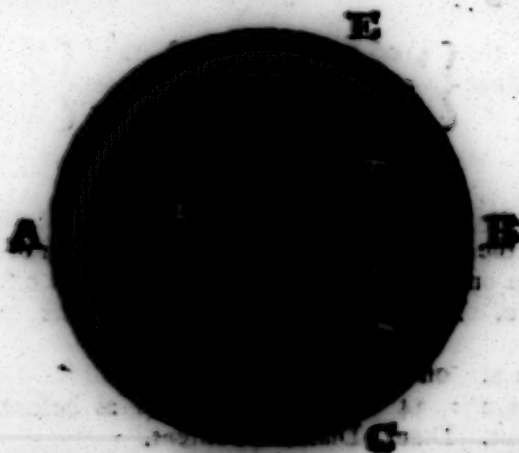
Facts. for a Square.

$$\frac{2}{3} \text{ of } \left\{ \begin{array}{l} .000365 \\ .002785 \\ .003399 \end{array} \right\} \text{ is equal } \left\{ \begin{array}{l} .000243 \\ .0018567 \\ .002267 \end{array} \right\} \left\{ \begin{array}{l} \text{Malt Bs.} \\ \text{Ale Gs.} \\ \text{Wine Gs.} \end{array} \right.$$

2. For Divisors the Proportion is

$$\text{as } \left\{ \begin{array}{l} 1 : 1.5 :: 2737.9 : 4106.99 \\ 1 : 1.5 :: 359.05 : 538.58 \\ 1 : 1.5 :: 294.12 : 441.18 \end{array} \right\} \text{ Div. } \left\{ \begin{array}{l} \text{M. bs.} \\ \text{A. gs.} \\ \text{W. gs.} \end{array} \right. \text{ for }$$

And in this Manner may be found Factors and Divisors for a Sphere in all the different Denominations mentioned in Page 49.



R U L E.

Cube the Diameter of the Sphere, and multiply or divide that Cube by the Factors, &c. above found, and the Product or Quotient will be the Content of the Sphere.

E X A M.

E X A M P L E.

Suppose the Diameter of the Sphere $A B C$ were 34 Inches $= A B$, what will its Content be in Ale Gallons? Vide last Figure.

O P E R A T I O N.

	Content.
$A B = 34$	538.58)39304.00(72.9 = $A G$
34	377006
136	.160340
102	107716
Sq. of $A B = 1156$	.526240
34	484722
4624	Rem. — .41518
3468	
Cube of $A B = 39304$	

The Proportion by the Lines d and e on the Rule.

on d .	on e .	on d .	on e .
As 1 :	.001856 ::	34 :	72.9 = as above.
Unity.	Factor.	Diam.	Content.

The Rule being thus set, is as a Table, for against any Diameter on d is the Content in A. G. on e ; the like for W. G. or M. B. For if Unity on d be set to the respective Factor on e , then against any Diameter on d is the Content in W. G. or M. B. upon e .

The same may be performed by the Lines c and d on the Rule; for if the Square Roots of the several Divisors be extracted, these Roots will be the Gage-points for a Sphere.

Thus the $\left\{ \begin{array}{l} 4106.99 \\ 538.58 \\ 411.18 \end{array} \right\}$ is $\left\{ \begin{array}{l} 64.1 \\ 23.2 \\ 21.0 \end{array} \right\}$ the $\left\{ \begin{array}{l} \text{M. B.} \\ \text{A. G.} \\ \text{pt. for W. G.} \end{array} \right\}$

And the Proportion will be

As the Gage-point on c is to the Diameter of the Sphere on c , so is the Diameter on d to the Content upon c . As for Example in the above Case.

on d .	on c .	on d .	on c .
As 23.2 :	34 ::	34 :	72.9 $A G$ as before.
Gage-pt.	Diam.	Diam.	Content.

In like Manner may the Content be found in any other Denomination, provided you make use of its proper Gage-point.

A R T.

A R T. VII.

Having the Altitude of the Frustrum of a Sphere BO , with the Diameter of the Base EC : To find the Altitude of the other Frustrum AO , or the Remainder of the Sphere's Diameter. See the last Figure.

R U L E.

Divide the Square of half the Diameter of the Frustrum's Base by the Altitude of the given Frustrum, and the Quotient will be the Altitude of the other Frustrum, or Remainder of the Sphere's Diameter.

E X A M P L E.

Let the Altitude of the Frustrum of the Sphere $BO = 9$ Inches $= BO$, and the Diameter of the Base $EC = 30$ Inches: To find the Altitude of the other Frustrum, or Remainder of the Diameter of the whole Sphere.

O P E R A T I O N.

$$\begin{array}{r}
 \frac{1}{2} EC = EO = 15 \\
 15 \\
 \hline
 75 \\
 15 \\
 \hline
 9)225(25 = \text{Alt. of other Frustrum} \\
 18 \qquad \qquad \qquad = AO. \\
 \hline
 45 \\
 45 \\
 \hline
 0
 \end{array}$$

A R T. VIII.

To find the Content of the lesser Frustrum of a Sphere by having its Diameter EC , and Altitude BO given. See the last Figure.

R U L E.

TO three Times the Square of half the Diameter of its Base add the Square of its Altitude; this Sum multiply by its Altitude, and the Product multiply or divide by the proper Factors, &c. for a Sphere in Page 75, and the Product or Quotient will be the Content required.

E X A M P L E.

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EXAMPLE.

Let the Dimensions be the same as in the last Article, viz. Diameter of the Base $BC = 30$ Inches, and the Altitude $AO = 9$ Inches: To find the Content in Ale Gallons.

OPERATION.

Square of $\frac{1}{2}$ the Diam. $BC = 225$	$AO = 9$	
3	9	
3 Times Sq. of $\frac{1}{2}$ the Diam. $= 675$	Sq. $AO = 81$	
Square of Altitude $= 81$		
Sum $= 756$		
Altitude $AO = 9$		
	Content.	
	538.58)6804.00(12.6 = A.	
	53858	Ga.
	141820	
	107716	
	.341040.	
	323148	
Remains $= .17892$		

A R T. IX.

To find the Content of the greater Frustrum of a Sphere, or the Part CAE . See the last Figure.

THIS may be found the same Way as the lesser Frustrum was in the last Article, or by the following Rule, which will also serve for either Frustrum.

R U L E.

Multiply the Square of half the Diameter by three Times the Altitude of the Frustrum, and to this Sum add the Cube of the Altitude; and this last Sum multiply or divide by the Factors, &c. for a Sphere in Page 75, the Product or Quotient will be the Content required.

EXAMPLE.

Let the Diameter of the Base BC be 30 Inches, and the Altitude AO be 15 Inches: To find the Content in Ale Gallons.

OFF.

OPERATION.

	Alt. $AO = 25$	
	25	
	—	
	125	
	50	
Square of $\frac{1}{2}$ the Diam. $EC = 225$	—	
3 Times the Altitude $AO = 75$	Sq. Alt. $= 625$	
	—	
	1125	
	1575	3125
	—	1250
Product —————	$= 16875$	
Cube of the Alt. —————	$= 15625$	C. Alt. $= 15625$
	—	
	538.58	32500.00
	323148	(60.3 = Cont. in Alc Ga.
	—	
	. 185200	
	161574	
	—	
Remains ———	. 23626	

PROOF.

The Content of the greater Frustrum $CAE = 60.3$
 The Content of the lesser Frustrum $EBG = 12.6$
 —————
 The Content of the whole Sphere ——— $= 72.9$

Which agrees with the Content of the whole Sphere, as found in Art. vi. of this Chapter; and in like Manner may the Content be found in any other Denomination mentioned in Page 49, if only you reduce the Factors, &c. for Cylinders to those for a Sphere, according to Art. vi. of this Chapter.

ART. X.

By having the Top and Bottom Diameters of the Frustrum of a Cone, to find intermediate Diameters at any assigned Depth.

RULE.

Divide the Difference between the Top and Bottom Diameters by the Frustrum's Depth, and the Quotient will be a common Factor to multiply any Depth by; the Product of which added to the Diameter of the lesser Base (if the Depth was taken from the lesser Base) will be equal to the Diameter at that Depth, or Distance from the Base. But if the Depth be taken from the greater Base, then

80 *Of finding the Contents of Solids.*

then subtract the Product of the common Factor, and Depth from the Diameter of the greater Base, and the Remainder will be equal to the Diameter at the Depth taken.

EXAMPLE.

Let the Conc's Frustrum be the same as that in Art. iv. of this Chapter, where the Diameter of the greater Base $cd = 38$ Inches, and the Diameter of the lesser Base $fg = 20.2$ Inches, and the Depth $ac = 21$ Inches: To find intermediate Diameters at every three Inches of its Depth.

OPERATION.

$$cd = 38.$$

$$fg = 20.2$$

$$ac = 21 \overline{) 17.8(.848} = \text{Com. Factor.}$$

168

100

84

160

168

Remains — .8

Factor.	Diff.	Les. Diam.	Diam.	
$3 \times .848 = 2.5$	$+$	$20.2 = 22.7$	at	3
$6 \times .848 = 5.1$	$+$	$20.2 = 25.3$	these	6
$9 \times .848 = 7.6$	$+$	$20.2 = 27.8$	Distts.	9
$12 \times .848 = 10.2$	$+$	$20.2 = 30.4$	from	12
$15 \times .848 = 12.7$	$+$	$20.2 = 32.9$	lesser	15
$18 \times .848 = 15.3$	$+$	$20.2 = 35.5$	Base.	18

If the Differences above had been subtracted from the greater Diameter the Remainders would have been the Diameters at the same Distances from the greater Base.

CHAR.

CHAP. IX.

Of gaging and fixing Victuallers open Utensils, &c.

IN this Chapter I shall shew the Method made use of by the Officers of Excise in gaging and finding the Area of Victuallers open Utensils, with the Manner of fixing of them in the Dimension Book; and also the Method of gaging all Sorts of By Tubs, &c. I shall first begin with the Mash Tun, and so proceed in the same Order as they must stand in the Dimension Book.

ART. I.

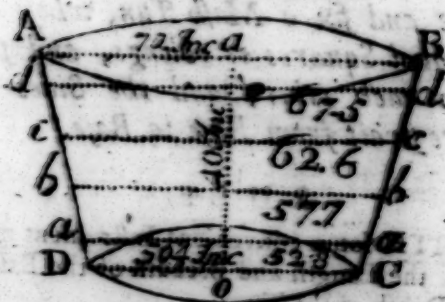
To gage and fix a Victualler's Mash Tun in Form of the Frustrum of a Cone, commonly called a Round Mash Tun. See the Figure below.

RULE.

1. **W**ITH a Dimension Cane, or Beams's Rule, find the Top and Bottom Diameters, and also the Depth of the Tun; then add the two Diameters together, and take half that Sum for a Mean Diameter, which will be near enough the Truth in Practice.

2. To find the Area at one Inch deep, square the Mean Diameter and divide that Sum by the Circular Divisor for a Mash Tun Gallon on Page 49, and the Quotient will be the Area at one Inch deep, which serves as a Mean for the whole Depth.

A ROUND MASH TUN.



EXAMPLE

Let the above Figure represent the Mash Tun whose Top Diameter $AB = 70$ Inches, the Bottom Diameter $DC = 50.4$ Inches, and Depth $AD = 6$ Feet.

82 *Of gaging, &c. Vintners open Utensils.*
 = 40 Inches, as found by the Dimension Cane:
 To find the Area at one Inch deep.

OPERATION.

Top Diameter A B — = 70
 Bottom Diameter D C — = 50.4
 Sum — = 120.4
 Half Sum — = 60.2 = Mean Diam.

120.4
 36,120

Circular Divisor = 289)3624.04(12.54 = Area in
 289 Gs.

734
 578

1560
 1445

1154
 1156

Remains — . . . 2

The Proportion by the Rule for the Area.

on D.	on C.	on D.	on C.
As 17	: 1	:: 60.2	: 12.54 = as above.
Circular	Unity.	Mean	Area.
Gage-pt.		Diam.	

The Area thus found, with the given Dimensions, are to be put into the Dimension Book, according to the Specimen exhibited in Chap. xiii. And in like Manner may the Area of any other Round Mash Tun be found.

A R T. II.

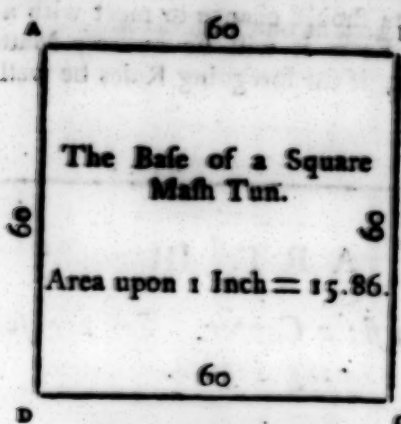
To gage and fix a Mash Tun, whose Bases are equal Squares or oblongs, and parallel to each other, and the Sides every where strait from Base to Base.

R U L E.

1. WITH some convenient Instrument measure the Length and Breadth of the Base, and also the Depth of the Tun:

2. Multiply the Length by the Breadth of the Base, and the Product multiply or divide by the Factors, &c. for a Square Mash Tun in Page 81 and the Product or Quotient will be the Area at one Inch deep.

E X A M P L E



EXAMPLE.

Let the above Figure represent the Base of a Square Mash Tun, whose Sides $AB = BC = CD = DA$; suppose I found with my Dimension Case $= 60$ Inches, and the Depth $= 30$ Inches: To find the Area in Mash Tun Gallons.

OPERATION.

Side $AB = 60$

Side $BC = 30$

Sq. Divisor $= 227 \mid 3600 (15.86 = \text{Area in Ale Gallons.}$

227
—
1330
1135
—
1950
1816
—
1340
1362
—

Remains — .22

The Proportion by the Rule.

on D. on C. on D. on C.
As 15.86 : 1 :: 60 : 15.86 = as above.
Sq gage- Unity. Side of Area.
point. Sq.

The Area thus found, with the given Dimensions, is put into the Dimension Book according to the Specimen exhibited in Chap. xiii. and it is done.

N. B. All Gages that are taken in Mash Tuns are cast up by multiplying the Area of one Inch by the Depth, whose Product is equal to the Content in Gallons.

Thus I have shewn how to gage and fix both round and square Mash Tuns, which are the most usual

84 *Of gaging, &c. Vessels open Utensils.*
usual Forms that these Kinds of Utensils are made
in: But if you should chace to meet with a Tun
of any other Form it will be an easy Matter to
gage and fix it, if the foregoing Rules be well un-
derstood.

A R T. III.

*To gage and fix a Copper. See the follow-
ing Figure.*

R U L E.

1. **O**BERVE whether or no the Copper hath
any considerable Fall in the Middle, or if
it hath a rising Crown; and if it hath neither of
them very considerable, then take the Depth of it
in the Middle between the Center and the Side, as
at B F.

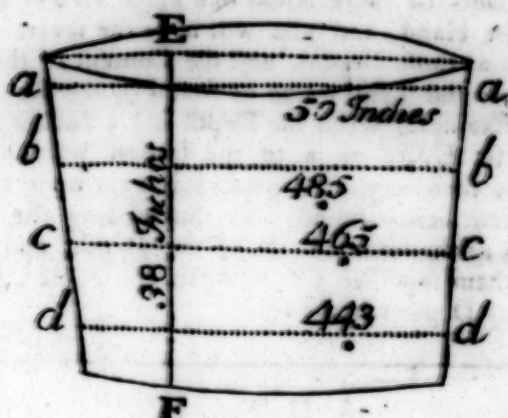
2. Take Mean Diameters between every six or
ten Inches. Thus on your *Brass*'s Rule or Di-
mension Cane, make Marks with Chalk at 3, 9,
15, or at 5, 15, 25 Inches, according to the
Depth of the Copper: Then set the Instrument
perpendicular to the Bottom of the Copper, and
at those several Marks on the Instrument mark
also the Sides of the Copper on each opposite Side,
and also at cross Diameters to the two former
Marks.

3. With your Instrument take Mean Diameters
at those Marks, by turning it round at those Places,
in order to see if the Diameters are equal every
Way, or not; which Mean Diameters enter on a
Piece of Paper as you take them, as also the Depth
of the Copper.

4. The next Thing is to find the Area at the
several Mean Diameters, which may be done by
Alg. ix of Chap. vii; or by the Table of Ale Areas.
The latter Method, I think, is preferable for fix-
ing of Utensils, as it saves the Trouble of finding
of the Area by the Pen, and you are sure to have
the Decimal true to two Places of Figures, which
you cannot be sure to have by the Sliding Rule,
altho' you may have a good one. Nevertheless it
will be proper to try them by the Rule, in order
to prevent any Mistake that may happen in the
taking them out of the Tables.

A COPPER.

A COPPER.



EXAMPLE.

Let the above Figure represent the Copper to be gaged and fixed, and suppose that I found the Dimensions of it by the Directions in this Article, as follows, *viz.* the Depth = $EF = 38$ Inches, the first Mean Diameter 5 Inches from the Bottom = $dd = 44.3$ Inches, the second at 15 Inches deep = $cc = 46.5$ Inches, the third at 25 Inches deep = $bb = 48.5$ Inches, and the fourth at 35 Inches deep = $aa = 50$ Inches; which Mean Diameters I set down as under, and turn to the Table of Ale Areas, and take out the Areas answering each respective Diameter, which I set against those Diameters, and then transfer the Whole into the Dimension Book according to the Specimen exhibited in Chap. xiii. and it is done.

	Diam.	Areas.	
Mean	$aa = 50$	$= 6.96$	as found by the Table of Ale Areas.
	$bb = 48.5$	$= 6.55$	
Diam.	$cc = 46.5$	$= 6.02$	
	$dd = 44.3$	$= 5.46$	

The lower Area 5.46 serves for the first 10 Inches from the Bottom, the second Area 6.02 serves for the second 10 Inches, *viz.* from 10 to 20 Inches, the third Area 6.55 serves for the third 10 Inches, *viz.* from 20 to 30, and the fourth Area 6.96 serves from 30 to 38, the last 8 Inches of the Copper's Depth: As for Example, at 15.5 Inches deep of this Copper, how many Gallons of Ale would it contain?

$$\begin{array}{r} 10 \times 5.46 = 54.6 \\ 5.5 \times 6.02 = 33.2 \\ \hline \end{array}$$

The Content at 15.5 Inches = 87.8 A. Galls.

When the Depth is 10 Inches the Trouble of Multiplication is saved for it is only removing the

86 Of gauging, &c. Vissnallers open Utensils.
Dot in the Area one Place farther to the Right Hand.
as $5.46 \times^4$ by 10 = 54.6, and so of any other. When
it is 20 or 30 Inches deep, you need only to remove
the Dots of those Areas one Place farther to the
Right Hand, and you will have the several Con-
tents at those Depths, and the Content of the odd
Inches and Tenths may be found by the Rule: As
for Example, when the Depth is 5.5 Inches above
10, set Unity on B to the second Area on A =
6.02, then against 5 5 on B is 33.2 on A = the
Content before found. By thus finding the Con-
tents of the several Parts of the Depth, and add-
ing them together, you will gain the whole Content
at any Depth whatsoever.

ART. IV.

To gage and fix an Under Back, whose Bases are Ovals or Ellipses, whether they be equal or unequal, provided they be parallel to each other.

RULE.

1. **W**ITH some convenient Instrument take the Transverse and Conjugate Diameters about the Middle of the Depth, and also the Depth, which is commonly about 10 or 12 Inches, and set them down on a Piece of Paper.

2. Find the Area on 1 Inch by Art. x. of Chap. vii. which, with the other Dimensions, transfer to the Dimension Book, according to the Specimen exhibited in Chap. xiii. and it is done.

EXAMPLE.

Let the Figure in Page 62 represent the Base of the Under Back, whole Transverse Diameter = 72 Inches, and Conjugate Diameter = 50 Inches, and suppose I find the Depth = 10 Inches, it is required to be fixed in the Dimension Book.

OPERATION.

Trans. Diam. $A B = 72$

Conj. Diam. $CD = 50$

$$\text{Circ. Div.} = \frac{359.05}{35905} \times 360000 (10.03 = \text{Area in A. G.})$$

..95000
107715

Remains — 12715

The Area is not quite 10.03, but is nearer to that than 10.02; and because we make use of but two Decimals in the Area we always take the second the nearest to the Truth. The Area thus found, with the other Dimensions, transfer to the Dimension Book, according to the Specimen exhibited in Chap. xiii. and it is done. ART.

ART. V.

To gage and fix a Back or Cooler.

DEFINITION.

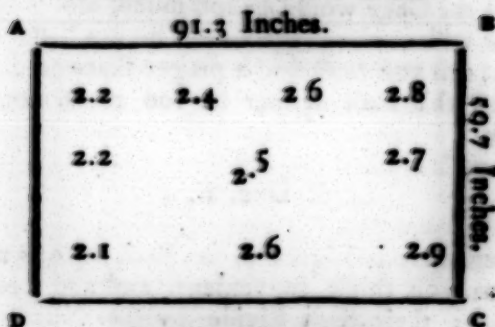
BACKS or Coolers are generally made in the Form of a Square, or an Oblong, and about 7 or 8 Inches deep.

RULE.

1. With a Dimension Cane, or some other convenient Instrument, take the Length and Breadth of the Back about the Middle of the Depth, which set down on a Piece of Paper.

2. Find the Area at one Inch deep by Art. i. and ii. of Chap. vii. which Area, with the Dimension taken, transfer to the Dimension Book, according to the Specimen exhibited in Chap. xiii. and it is done.

A BACK or COOLER.



EXAMPLE.

Let the above Figure represent the Bottom of the Cooler, and suppose I found the Length of it $AB = 91.3$ Inches, and the Breadth $BC = 59.7$ Inches: To find the Area, and fix it in the Dimension Book.

OPERATION.

Length $AB = 91.3$

Breadth $BC = 59.7$

6391
8217
4565

Square Div. $= 282)5450,61(19.33 = \text{Area in Ale Galls.}$

282
2630
2538
—
926
846
—
804
846
—

Remains — 45
G 4

The

88 *Of gaging, &c. Vintners open Urnalls.*

The Area thus found, with the Length and Breadth, is transferred to the Specimen of a Dimension Book in Art. of Chap. which was required.

It will not be amiss to try those Areas by the Rule before you enter them into the Dimension Book, in order to prevent any Mistake that may happen in the Operations.

A R T. VI.

To find a Dipping Place in a Back or Cooler.

BECAUSE Backs are not set level, but rather sloping for the Conveniency of the Wort's running off; therefore if you should dip in too deep a Place the Gage would be too much, and if in too shallow a Place the Gage would be too little. To prevent this you must find a proper Place to dip always at, that shall neither be too much nor too little.

R U L E.

When there is Liquor in the Back take as many Dips as you think convenient, and add them all together; their Sum divide by the Number of Dips taken, and the Quotient will be a Mean Depth. When this is done, try in several Places of the Back till you find a Dip that will answer your Mean Depth; which when found, mark the Side of the Back right against that Place for a constant Dipping Place.

E X A M P L E.

Suppose I take 10 Dips in the Back A B C D (vide last Figure) at the several Places where the Depths are set in the Figure, then if they are added together they will stand as in the Margin, and their Sum will be = 25, which divided by 10, or which is all one, remove the Dot one Place farther to the Left Hand, gives 2.5 for a Mean Depth, which by Trials I find will answer at g; against which I set a Mark for my constant Dipping Place, and it is done.

	Depths.
2.2	
2.3	
2.1	
2.6	
2.8	
2.7	
2.4	
2.6	
2.9	
2.8	

10)25.0(2.5

A R T.

ART. VII.

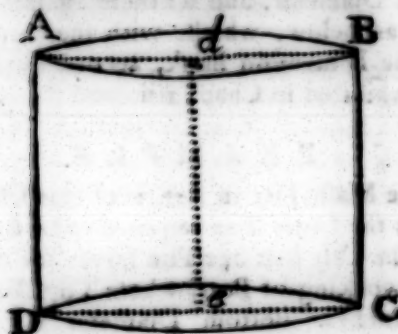
To gage and fix a Guile Tun, or Round, in Form of a Cylinder.

RULE.

1. **W**ITH some convenient Instrument find the Diameter of the Tun, and also the Depth, which set down on a Piece of Paper.

2. With that Diameter look into the Table of Ale Areas and find the Area answering thereto, which Area, with the Diameter and Depth, transfer to the Dimension Book, according to the Specimen exhibited in Chap. xiii. and it is done.

A ROUND GUILLE TUN.



EXAMPLE.

Let the Figure above represent the Tun to be fixed, and suppose I found $AB = DC = 50$ Inches, and the Depth $de = 40$ Inches: To find the Area, and fix it in the Dimension Book.

OPERATION.

By the Table of Ale Areas I find the Area answering 50 Inches = 6.96, which, with the Depth and Diameter, I put into the Dimension Book, according to the Specimen exhibited in Chap. xiii. which was required.

Note, All Depths which are taken in those Utensils that are fixed upon one Area must be multiplied by that Area, in order to find the Content of the Liquor in it at any Time. As for Example, suppose I had a Gage of Ale in this Tun of 17.3 Inches deep, what is the Content of that Gage?

Operation by Rule.

ON B.	ON A.	ON B.	ON A.
As 1 :	6.96 ::	17.3 :	120.4
Unity.	Area.	Depth.	Content.

A R T. VIII.

To gage and fin a Guile Tun in the Form of the Frustrum of a Cone.

R U L E.

1. **W**ITH some convenient Instrument take Mean Diameters between every 10 Inches, in the same Manner as the Copper was done in Art. iii. of this Chapter, and also the Depth; or if it be the Frustrum of a Right Cone, take the Top and Bottom Diameters, and by Art. x. of Chap. viii. find Mean Diameters between every 10 Inches, which set down on a Piece of Paper.

2. With those Mean Diameters, seek in the Table of Ale Areas for the respective Areas answering each Diameter, and set them against their Diameters, as below, which, with the Depth, transfer to the Dimension Book, according to the Specimen exhibited in Chap. xiii. and it is done.

E X A M P L E.

Let the Mash Tun in Art. i. of this Chapter also represent the Guile Tun required to be fixed (which is often the Case that one Tun serves for both mashing and working of Beer) whose Top Diameter $aa = 70$ Inches, Bottom Diameter $bc = 50.4$ Inches, and Depth $ee = 40$ Inches: To find Mean Diameters between every 10 Inches, and to fix it in the Dimension Book.

O P E R A T I O N.

$$AB = 70.$$

$$BC = 50.4$$

$$40 \overline{) 19.6149} = \text{Com. Factor.}$$

$$160$$

$$360$$

$$360$$

•

Factor. Diff. B.diam. M.diam. Areas.

$$\begin{array}{l} 5 \times .49 = 2.4 + 50.4 = 52.8 \text{ aa} \\ 15 \times .49 = 7.3 + 50.4 = 57.7 \text{ bb} \\ 25 \times .49 = 12.2 + 50.4 = 62.6 \text{ cc} \\ 35 \times .49 = 17.1 + 50.4 = 67.5 \text{ dd} \end{array} \left\{ \begin{array}{l} 7.76 \\ 9.27 \\ 10.88 \\ 12.70 \end{array} \right\}$$

Thus have I found the Mean Diameters in the Middle between every 10 Inches, as above, and the Areas answering each Diameter, as per the Tables of Ale Areas, are set against them, which, with the Depth, I have placed in the Specimen of a Dimension Book exhibited in Chap. xiii. which was required.

A R T.

ART. IX.

To gage and fix a Guile Tun whose Bases are ellip'ical or oval, but unequal, called a Cylindroid.

R U L E.

1. **M**AKE Marks with Chalk at the Extremes both of the Transverse and Conjugate Diameters, according to the Directions in Art. iii. of this Chapter, in the Middle between every 10 Inches of the Depth, and with the Dimension Cane or Branan's Rule measure the Transverse and Conjugate Diameters at those Marks, beginning at the lower Base; take also the Depth, which Dimensions set down on a Piece of Paper, as below.

2. By Art. x. of Chap. vii. find the Areas of the several Diameters, and set them down, as under, against their respective Diameters, which, with the Depth, transfer to the Dimension Book, according to the Specimen exhibited in Chap. xiii. and it is done.

E X A M P L E.

Suppose I found the Depth of the Tun = 30 Inches, and the Transverse Diameter at 5 Inches deep = 42.2 Inches, and the Conjugate Diameter = 34.2 Inches; at 15 Inches deep the Transverse Diameter = 43.4, and the Conjugate = 36.5; and at 25 Inches deep the Transverse Diameter = 44.4, and the Conjugate = 38.4 Inches: To find the several Areas, and to fix the same in the Dimension Book.

The Proportion for the Areas by Pen or Rule is

$$\text{as } \left\{ \begin{array}{l} 359.05 : 44.4 :: 38.4 : 4.75 \\ 359.05 : 43.4 :: 36.5 : 4.41 \\ 359.05 : 42.2 :: 34.2 : 4.02 \end{array} \right\} \text{ at } \left\{ \begin{array}{l} 25 \\ 15 \\ 5 \end{array} \right\} \text{ Inches.}$$

Circ. Div. Transf. Conj. Areas.

	Diameters.	Areas.	
thus I have found if the	Transf. = 44.4	} = 4.75	which Areas and Diama. with the Depths, are transferred to the Speci- men of a Dimension Book exhibited in Chap. xiii. which was required.
	Conj. = 38.4		
	Transf. = 43.4	} = 4.41	
	Conj. = 36.5		
	Transf. = 42.2	} = 4.02	
	Conj. = 34.2		

ART.

ART. X.

To gage and fix a Guile Tun made of a Hog-head, or Pipe, standing upon one Head, the other being cut off. See the Figure following.

R U L E.

1. **F**IND Mean Diameters in the Middle between every 6 or 10 Inches, according to the Directions in Art. iii. of this Chapter, for the Copper. If the Tun is large, and the Diameters differ much, you may find Diameters in the Middle of every 4 Inches; which Diameters, and also the Depth, set down on a Piece of Paper.
2. With these Diameters find their respective Areas in the Table of Ale Areas and set them against their Diameters, as below, which, with the Depth, transfer to the Dimension Book, according to the Specimen exhibited in Chap. xiii. and it is done.

A GUILLE TUN.



EXAMPLE.

Let the above Figure represent the Tun to be gaged and fixed, and suppose I found the Depth of the Tun $AF = 24$ Inches, and the Diameter aa , 3 Inches from the Bottom, $= 24$ Inches, the Diameter bb , 9 Inches from the Bottom, $= 25$ Inches, the Diameter cc , 15 Inches from the Bottom, $= 26.5$ Inches, and the Diameter dd , 21 Inches from the Bottom, $= 24.9$ Inches: To find the several Areas, and fix it in the Dimension Book.

OPERATION.

By the Tables of Ale Areas I find the Areas answering each Diameter as follows.

	Diams.		Areas.	
against	$dd = 24.9$	are	$\left\{ \begin{array}{l} 1.73 \\ 1.96 \\ 1.74 \\ 1.60 \end{array} \right\}$	in the
the	$cc = 26.5$	those		Table of
Diams.	$bb = 25$	Areas		Ale
	$aa = 24$			Areas.

It

It will not be amiss to try your Work by the Rule, by setting Unity on *c* to the Circular Ale Gaugepoint on *a*, then, if there is no Mistake made, against the several Distances on *a* are the Areas before found on *c*, which, with the Depth, I have transferred to the Specimen of a Dimension Book in Chap. xiii. as was required.

Note, all Depths that are taken in this Tun must be multiplied by their respective Areas, *viz.* if 6 Inches, or under, by the lower Area; that Part of the Depth which is between 6 and 12 Inches by the second Area; that between 12 and 18 Inches by the third Area; and that Part between 18 and 24 by the upper Area: Which several Contents of the Parts added together will be the Content of the whole Tun.

For Example, suppose the Depth of the Liquor in this Tun was 20 Inches, how many Gallons would it contain?

OPERATION.

$$\begin{array}{rcl} 6 \times 1.60 & = & 9.6 \\ 6 \times 1.74 & = & 10.44 \\ 6 \times 1.96 & = & 11.76 \\ 2 \times 1.73 & = & 3.46 \end{array}$$

$$\text{Depth } 20 \quad = \quad 35.26 = \text{Content.}$$

A R T. XI.

To gage and fix a Gaile Tun, whose parallel Bases are both Squares or Parallelograms, but unequal.

R U L E.

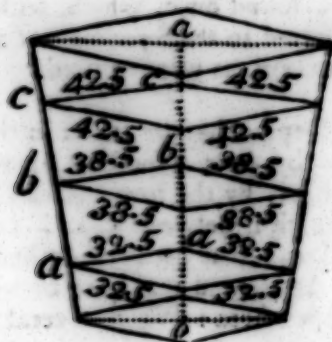
1. **W**ITH some convenient Instrument find the Depth of the Tun, and also the Length and Breadth in the Middle between every 10 Inches, by the Directions in Art. iii. of this Chapter, which set down on a Piece of Paper.

2. By Art. i. or ii. Chap. vii. find the Areas at the several Depths, and set them down against their respective Dimensions, as in the following Work, which, with the Depth, transfer to the Dimension Book, according to the Specimen of a Dimension Book in Chap. xiii. and it is done.

E X A M P L E.

Suppose the following Figure to represent the Tun, which is the Frustum of a Square Pyramid, whose Depth as I found = 30 Inches; and the Length of the Side *ac*, 5 Inches from the Bottom = 32.5; and at 15 Inches from the Bottom, the Side *de* = 36.5

96 Of gaging, &c. *Vitruallers open Utensils.*
 = 38.5 Inches; and at 25 Inches deep, the Side
 = 42.5 Inches: To find the Areas at the several
 Depths, and to fix the same in the Dimension
 Book.



OPERATION.

Side <i>aa</i> 32.5	Side <i>bb</i> 38.5	Side <i>cc</i> 42.5
32.5	38.5	42.5
1625	1925	2125
650	3080	850
Sq. 975	1155	1700
Div. — Area	— Area	— Area
282)1056.25(3.74	1482.25(5.25	1806.25(6.41
846	1410	1692
2102	.722	1142
1974	564	1128
1285	1585	.145
1128	1410	282
Rem. .157	.175	137

Thus have $\left\{ \begin{array}{l} L \ 32.5 \\ B \ 32.5 \end{array} \right\}$ the $\left\{ \begin{array}{l} 3.74 \\ 5.25 \\ 6.41 \end{array} \right\}$ Which Sides
 I found $\left\{ \begin{array}{l} L \ 38.5 \\ B \ 38.5 \end{array} \right\}$ Areas will be $\left\{ \begin{array}{l} 3.74 \\ 5.25 \\ 6.41 \end{array} \right\}$ and Areas,
 that if the $\left\{ \begin{array}{l} L \ 42.5 \\ B \ 42.5 \end{array} \right\}$ will be $\left\{ \begin{array}{l} 3.74 \\ 5.25 \\ 6.41 \end{array} \right\}$ with the
 Sides are $\left\{ \begin{array}{l} L \ 42.5 \\ B \ 42.5 \end{array} \right\}$ be $\left\{ \begin{array}{l} 3.74 \\ 5.25 \\ 6.41 \end{array} \right\}$ Depth, are
 men of a Dimension Book exhibited in Chap.
 xiii. as was required.

In this Manner may *Vitruallers Utensils* of any
 other Form be gaged and fixed, be they ever so ir-
 regular, by taking more or less Mean Diameters,
 or Mean Lengths and Breadths of the Sides, accord-
 ing to the Regularity or Irregularity of the Vessel.
 I have purposely omitted to give an Example of
 a Square Guile Tun, whose Bases are equal and pa-
 rallel, and Sides every where streight Lines, be-
 cause that is done in the same Manner as the
 Back, or Cooler, in Art. v. of this Chapter; only
 the Depth must be taken, and entered over the Sides
 of the Square in the Dimension Book; which is
 not done in fixing of the Back, or Cooler.

A R T.

A R T. XII.

To gage all Sorts of By Tubs.

D E F I N I T I O N.

BY Tubs are such as are not fixed upon Areas in the Dimension Book, but require other Dimensions to be taken besides the Depth, in order to find the Content, *viz.* the Length, and Breadth or Mean Diameter.

1. To gage such Sort of Vessels, whether they be cylindrical, conical, or elliptical.

R U L E.

Take the Depth of the Liquor in the Vessel, and also a Mean Diameter, by laying the Gaging Rod over the Surface of the Liquor. If the Top and Bottom Diameters are unequal you must guess at a Mean Diameter; or if the Tub be elliptical lay the Gaging Rod across between the Transverse and Conjugate Diameters, and guess at a Mean Diameter that will reduce the same to a Cylinder; which Mean Diameter set down in your Book over the Depth, with the Character or Quality of the Liquor contained in the Tub over it.

2. To gage By Tubs that have Squares or Parallelograms for their Bases, whether they be equal or unequal.

R U L E.

Take the Mean Length and Breadth of the Tub, and also the Depth of the Liquor in it; which set down in your Book, *viz.* the Depth under, next the Breadth with the Letter B. against it for Breadth, and above that the Length with the Letter L. against it for Length, and lastly set the Character over it, and it is done

In some Places it is customary to cool the Worts in Brass Pans or Kettles. In such Vessels the Diameter is found as above; but the Depth must not be taken in the Middle, but about four or five Inches from the Side, according to the Size of it, because these Kind of Vessels have a Fall in the Middle.

Sometimes Worts are cooled in Bowls. In such Cases take the Diameter of the Liquor's Surface, and also the Depth in the Middle of the Bowl; but set down only half that Depth in your Book, which will answer pretty near the Truth in Practice; but Experience will be the best Guide in such Cases.

A R T. XIII.

To deduct the Heat out of warm Worts in casting up the Gages.

IT has been found by Experience that 10 Gallons of hot Wort will be but nine Gallons when cold: Therefore, by Act of Parliament, there should be an Allowance made of one Gallon in every ten Gallons gaged hot. To do this observe the following Directions:

R U L E.

This Deduction may be made either by Subtraction or Multiplication.

1. By Subtraction set down the Number of warm Gallons twice; but the under Number must be set one Figure farther to the Right Hand, or, which is the same, remove the Dot of the Subtrahend one Place farther to the Left Hand, and it is divided by 10, and this tenth Part subtracted from the Number of warm Gallons will leave the Quantity of Wort to be charged.

2. By Multiplication. Multiply the Number of warm Gallons by .9 a Decimal, and the Product will be neat Gallons.

E X A M P L E.

Suppose a Gage of warm Wort were 45 Gallons, what must be charged?

O P E R A T I O N.

By SUBTRACTION.

$$\begin{array}{r} 45 \\ 4.5 \\ \hline 40.5 = \text{neat Gs.} \end{array}$$

By MULTIPLICATION.

$$\begin{array}{r} 45 \\ .9 \\ \hline 40.5 = \text{neat Gs.} \end{array}$$

Thus you see that by either Method the Answer is the same: But this may be performed by the Rule without knowing the Number of warm Gallons at all: But to do this there must first be a Divisor found.

And as Divisors are the Reciprocals of Multipliers, it is only dividing Unity by .9, and the Quotient will be a proper Divisor to effect the same. For Example.

$$.9)1.0(1.11 = \text{Divisor.}$$

$$\begin{array}{r} 9 \\ \hline .10 \\ 9 \\ \hline .10 \\ .9 \\ \hline \text{Remains} = 1 \end{array}$$

Thus

Of deducting the Heat out of warm Worts 97

Then the Proportion will be for the Example before,

on B.	on A.	on B.	on A.
As 1.11	: 1	:: 45	: 40.5 = as before.
Divisor.	Unity.	Warm	Neat
		galls.	galls.

But when the Number of warm Gallons are not known, and you have the Depth of the Liquor in a fixed Utensil, and also the Area of that Depth given, then the Proportion is

As the Divisor is to the Area, so is the Depth to the Content in neat Gallons. As for Example, suppose the Depth of the Liquor was 9 Inches, and the Area belonging to that Depth was 5 Gallons, how many neat Gallons would it contain?

OPERATION.

on B.	on A.	on B.	on A.
As 1.11	: 5	:: 9	: 40.5 = neat Galls.
Divisor.	Area.	Depth.	Content.

For $9 \times 5 = 45$, and one Tenth of 45 being deducted from it leaves 40.5, as above; and so of any other. Therefore it will be necessary to make a Mark at 1.11, either upon the Line A or B, for the Conveniency of casting up of warm Gages.

If the warm Worts are in Circular By-Tubs, and the Depth under Diameters, then, instead of the Gage-point 18.95 on the Line D, make use of 20, to which set the Depth on c, and against any Diameter on d is the Content in neat Gallons on c. As for Example, suppose the Diameter of a By Tub = 30 inches, and the Depth of warm Wort in it = 12 Inches, how many neat Gallons does it contain?

OPERATION.

on D.	on C.	on D.	on C.
As 20	: 12	:: 30	: 27 = neat Gallons.
Warm gage-	Depth.	Diam.	Content.
point.			

The Heat is deducted out of common Brewers warm Gages in a different Manner, by reason of the different Denominations in the given Number of Barrels, &c. Therefore to do this observe the following Directions: For every 10 Barrels set down 1 under the Barrels, and what remains above 10 Barrels reduce into Firkins, and add the Firkins (if any) to that Sum; and for every 10 Firkins set down 1 under the Firkins, and what remains above 10 Firkins reduce into Gallons, and

98 Of deducting the Heat out of warm Worts.

their Sum to the Gallons, and for every 10 Gallons set down 1 under the Gallons; then subtract the under Number thus found from the given Number of Barrels, &c. above it, and the Remainder will be equal to the neat Gallons. For Example,

Let the warm Barrels, &c. = 16 : 3 : 6. To find the neat Barrels.

B. F. G.

1 : 2 : 6.5

15 : 0 : 8 = neat Barrels.

In the above Example I set down 1 for the 10 Barrels under Barrels, and reduce the 6 Barrels into Firkins = 24, to which I add 3 = 27 F. then for the 20 Firkins I set down 2 under 20 Firkins; and reduce the other 7 into Gallons = 59.5 to which I add 6, which = 65.5. for which I set 6.5 under the Gallons, and subtract the lower Number from that above, and it is done.

EXAMPLE II.

Let the warm Barrels, &c. = 86 : 0 : 4 to find the neat Barrels, &c.

B. F. G.

8 : 2 : 4

77 : 2 : 0 = neat Barrels.

EXAMPLE III.

Let the warm Barrels, &c. = 9 : 3 : 0. To find the neat Gallons.

B. F. G.

0 : 3 : 7.5

8 : 3 : 1 = neat Barrels.

And in this Manner may the Heat be deducted out of any Number of Barrels, &c. whatsoever.

A R T. XIV.

To find new Gage-points.

WHEREAS it may happen sometimes in Business, that when one of the given Numbers is set to the Gage-point the other given Number will fall off the Rule. To remedy this there must be new Gage-points found.

Thus,

Set Unity on c to the old Gage-point on u, and against the other 1 on c is the new Gage-point on u.

EXAMPLE

EXAMPLE I.

Suppose it were required to find a new Gage-point to 18.95 the old Ale Gage-point.

OPERATION.

Set Unity on c to 18.95 on d, and against the other 1 on c is 59.92 the new Gage-point on a for circular Ale Gallons.

EXAMPLE II.

To find a new Gage-point for Wine Measure.

OPERATION.

Set Unity on c to 17.15 on d, and against the other 1 on c is 54.22 on a, the new Gage-point for circular Wine Gallons.

In like Manner may any other new Gage-point be found to any of the old Gage-points in the Table Page 49.

These second Gage-points are the Square Roots of the Divisors multiplied by 10. As for Instance, the circular Divisor for Ale Gallons = 359.05, which multiplied by 10 = 3590.5, whose Square Root = 59.92 the new Gage-point for Ale Gallons, as before found; and 394.12 the circular Divisor for Wine Gallons, multiplied by 10 = 3941.2, whose Square Root = 54.22 the new Gage-point for Wine Gallons, as before found; and so for any other new Gage-point.

The Use of the new Gage-point.

Suppose the Depth of a Vessel be = 8 Inches, and the Diameter = 9 Inches, what is the Content in Ale Gallons?

OPERATION.

on d. on c. on d. on c.

As 59.92 : 8 :: 9 : 1.8 = Ale Gallons.
New Ale Depth. Diam. Content,
Gage-pt.

But if the Content were required in Wine Gallons,

on d. on c. on d. on c.

As 54.22 : 8 :: 9 : 2.75 = Wine Gallons.
New wine Depth. Diam. Content.
Gage-pt.

C H A P. X.

THIS Chapter treats of gaging and inching, &c. of all Sorts of common Brewers open Utensils according to the common Methods made use of by the Officers of the Excise; and from thence is deduced a little Table Book, which is placed at the latter End in the same Form as if it was a real Table Book, which may serve as a Precedent for young Officers to apply to whenever they may have Occasion; for it may sometimes so happen that a Brew-house may be erected in a Division where there was none before.

I shall begin with the Mash Tun, and so proceed in that Order as they must stand in the Table Book.

A R T. I.

To gage and inch a Mash Tun whose parallel Bases are equal Squares or Parallelograms, and its Sides every where strait Lines.

R U L E.

1. **W**ITH some convenient Instrument take the Length and Breadth, and also the Depth of the Tun.

2. By Art. ii. Chap. ix. find the Area upon one Inch deep, and multiply that Area by the Depth and the Product will be the Content of the Tun in Gallons.

3. Reduce both the Area and the whole Content of the Tun into Quarters, Bushels and Gallons.

4. To find the Content upon every Inch of the Tun's Depth add the Area of the first Inch reduced to itself; and the Sum will be the Content at 1 Inch deep; to this Sum add the Area of the first Inch, and you will have the Content at 2 Inches deep; and in this Manner keep continually adding the Area of 1 Inch to the Content of the 1st, 2d, 3d, 4th, 5th, 6th, &c. Inches, till you arrive at the Depth of the whole Tun; which last will be equal to the Content of the whole Tun if the Work be done right, if otherwise not.

E X A M P L E.

Suppose I found the Depth of a Square Mash Tun = 30 Inches, the Length of the Bases =

Of gauging and inching C. Brewsters, &c. 102.
 119 Inches, and the Breadth = 102 Inches: To
 find the Content of the whole Tun, and also the
 Content at every Inch of its Depth.

OPERATION.

Length of Bases = 119

Breadth of Bases = 102

238

1190

Sq. Div. = 227)12138(

1135

53.47 = Area at 1
 30 = Depth.

.788 8)1604.10 = Cont. in

8)53.47 = Area in 681

Galls.

8)200;4.1 = Cont. in

6:5.47 = Area in 1070

Busbels, &c.

q. b. g. Busbels.

908

25:0:4.1 = Cont.

in Quarters, &c.

1620

1589

Remains — .31

Thus I have found the Content of the whole
 Tun = 25 Qrs. 0 Bs. 4.1 Ga. and the Area at
 1 Inch deep = 0 Qrs. 6 Bs. 5.47 Ga. the next
 Thing to be done is to find the Content at every
 intermediate Inch of the Depth, which is done
 thus,

Q. B. G.

0 : 6 : 5.47 = Content at 1 Inch.

0 : 6 : 5.47

1 : 5 : 3.14 = Content at 2 Inches.

0 : 6 : 5.47

2 : 4 : 0.61 = Content at 3 Inches.

0 : 6 : 5.47

3 : 2 : 6.08 = Content at 4 Inches.

0 : 6 : 5.47

4 : 1 : 3.55 = Content at 5 Inches.

And in this Manner I proceeded till I came to
 30 Inches, the whole Depth of the Tun, which I
 find to be 25 Qs. 0 Bs. 4.1 Ga. = the Content
 before found, which proves the Work to be done
 right. The several Contents thus found are
 placed in the Table Book Page 115, against their
 respective Inches throughout the whole Depth.
 The Dimensions and Area at one Inch, with the
 Content, are placed in the Front of the Table
 Book in Page 114, which was required.

In filling up of the Table I have rejected the odd
 Tens if they were under 5, and when they

the Of gaging and inching C. Brewster, 1740
 exceeded 5 I have added one more to the even
 Gallons.

If the Tun to be inch'd had been a Cylinder the Diameter must have been squared, and that Square divided by the circular Divisor for Mash Tuns in Page 49, and the rest of the Work would be just the same as for the Square Mash Tun; or if it had been the Frustrum of a Cone, then I should find Mean Diameters in the Middle between every 6 or 10 Inches, and then have found the Areas of these Diameters, which Areas added for every respective Inch to which they belong, will give the Content at every Inch of the Tun's Depth, as will appear more clearly by the following Example of the Copper.

ART. II.

To gage and inch a Copper with a rising Crown, and make an Allowance for the same. See the following Figure.

EXAMPLE.

LET the Figure $A B C D$ represent the Copper to be gaged and inch'd.

A COPPER.



RULE.

1. Take a small Cord or Thread, and fasten one End of it at a , and extend the other End to the opposite Side of the Copper at c , where make it fast; then with some convenient Instrument find the greatest Depth of it, which is $= a o = o c$; that is the nearest Distance from the Thread to the Bottom at o , and suppose it $= 30$ Inches.

In like Manner take the least Depth of the Copper at e , which is the nearest Distance from the Thread to the Top of the Crown at e , which suppose to be $= 27$ Inches, then $30 - 27 = 3$ Inches, the Height of the Crown.

2. To find the Diameter at the Bottom of the Crown, measure $A B$ the Top Diameter, which sup-

pole = 90 Inches; then hold a Thread with a Plummert at the End of it, so as that the Plummert may hang over *b* or *c*; then measure the Distance between the Edge of the Copper and the Place where the Threads cut each other = *aa* = *ea*, which suppose = 5 Inches, and $5 + 5 = 10$ subtracted from the Top Diameter = 90 leaves $80 = dc$, the Diameter at the Bottom of the Crown.

3. To find the Content of the Liquor required to cover the Crown, measure the Diameter *bi*, which touches the Top of the Crown, and suppose it = 82 Inches; then by having the Top and Bottom Diameters, and the Altitude of the Frustrum of a Cone, or the Part *bicd*, the Content of that Part may be found by Art. iv. Chap. viii. from which the Content of the Crown must be subtracted.

4. The Content of the Crown may be found by Art. viii. and ix. of Chap. viii. as the Frustrum of a Sphere; or thus, Multiply the Area of the Bottom Diameter = *dc* by half the Altitude of the Crown, and the Product will be nearly equal to the Content of the Crown, which subtracted from the Part *bicd* will leave the Quantity of Liquor required to cover the Crown.

Operation for finding the Quantity of Liquor to cover the Crown.

Diam. <i>bi</i> = 82	} Areas.	18.727	Area of the
Diam. <i>d</i> = 80		17.825	Diam. <i>dc</i>
Mean Diam. = 80.9		18.23	= 17.825
			$\frac{1}{2}$ Alt. = 1.5
The Content of the Part <i>bicd</i>	} =	54.782	
Content of the Crown		26.7375	
Content of Liquor to cover the Crown	} =	28.0445	26.7375

But the Content of the Liquor to cover the Crown may be found otherwise, viz. From the Area of the Diameter at the Top of the Crown = *b*; subtract $1 \frac{1}{3}$ of the Area of the Crown's Height, and the Remainder multiply by half the Height of the Crown, whose Product will be equal to the Number of Gallons to cover the Crown; or it may be found by Measure (which is the most practical Method) without finding the Dimensions of the Crown at all, by pouring in Water from a Gallon, or some other known Measure, till the Crown is covered.

At the same Time take Mean Diameters between every 6 or 10 Inches, beginning from the Top of the Copper towards the Bottom: As suppose in this Example I found the first Mean Diameter at 5 Inches from the Top = 88 Inches, at 15 Inches from the Top = 85.5 Inches, and at 25 Inches from the Top = 82.5 Inches, as they are set down in the second Column of the following Table.

By the Table of Ale Areas I find their respective Areas as they are set against these Diameters in the third Column; and the Contents of the several Parts of the Depth in Gallons are placed in the fourth Column; which several Contents are reduced to Barrels, &c. and placed in the three last Columns.

Pts. of Depth.	Diam.	Area.	Content in Gallons.	Content in		
				B.	F.	G.
10	88.0	21.568	215.680	6	1	3.180
10	85.5	20.960	209.600	5	3	1.100
07	82.5	18.956	132.692	3	3	5.192
03	To cover the Crown.		23.044	0	3	2.346
30	Content of Copper.		580.016	17	0	2.016

The two upper Contents in Gallons are found by removing the Dot in the Areas one Place farther to the Right Hand; the last Content is found by multiplying its correspondent Area by the Depth 7.

The next Thing is to find the Content upon every Inch of the Copper's Depth.

R U L E.

From the whole Content of the Copper reduced into Barrels, &c. subtract the Area of the first 10 Inches, and the Remainder will be the Content when one Inch is dry; and so continue subtracting that Area from the Remainder until 10 Inches are dry, and you will gain the Contents of the first 10 dry Inches. See the Work.

OPERATION.

OPERATION.

Whole	Continued.	Continued.
cont. R.F. G.	10=10:2:7.336	4:2:7.736
=17:0:2.016	Note 0:2:3.360	Note 0:2:1.956
Sub. 0:2:4.568		
	11=10:0:3.976	21=4:0:5.780
1=16:1:5.948	0:2:3.360	0:2:1.956
0:2:4.568		
	12=9:2:0.616	22=3:2:3.824
2=15:3:1.380	0:2:3.360	0:2:1.956
0:2:4.568		
	13=8:3:5.756	23=3:0:1.868
3=15:0:5.312	0:2:3.360	0:2:1.956
0:2:4.568		
	14=8:1:2.396	24=2:1:8.412
4=14:2:0.744	0:2:3.360	0:2:1.956
0:2:4.568		
	15=7:2:7.536	25=1:3:5.456
5=13:3:4.676	0:2:3.360	0:2:1.956
0:2:4.568		
	16=7:0:4.176	26=1:1:4.500
6=13:1:0.108	0:2:3.360	0:2:1.956
0:2:4.568		
	17=6:2:0.816	27=0:3:2.544
7=12:2:4.040	0:2:3.360	
0:2:4.568		Crown
	18=5:3:5.956	0:3:2.544
8=11:3:7.972	0:2:3.360	
0:2:4.568		Rem.
	19=5:1:2.596	=0:0:0.000
9=11:1:3.404	0:2:3.360	
0:2:4.568		
	20=4:2:7.736	
10=10:2:7.336	6:1:3.180	
6:1:3.180	5:3:8.100	
17:0:2.016	17:0:2.016	

When I have finished the Subtraction with the first Area, which is at the 10th dry Inch, I add the Content of the first 10 Inches to the Remainder, in order to prove my Work, whose Sum, if right, will be equal to the Content of the Copper, as before found.

Then I take the second Area and subtract it from the last Remainder, and so continue to do till I come to the 20th dry Inch, and then to prove my Work I add the Contents of the first and second 10 Inches to the last Remainder, whose Sum, if right, will be also equal to the whole Content of the Copper.

Lastly, I take the third Area and subtract it from the last Remainder, and so proceed till I come to the 37th dry Inch, whose Remainder, if the Work

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 Work be done right, will equal the Quantity of
 Liquor to cover the Crown, as before found. Thus
 having finished the Subtraction I transfer the several
 Contents to the nearest even Gallon to the Table
 Book in Page 115, and it is done.

I have been the longer on this Article in order
 to render the Affair of inching, &c. not only of
 the Copper, but also all other Sorts of Utensils that
 stand upon several Areas, as plain as possible; for
 the Content at every Inch of the Depth is found
 after the same Manner.

In adding and subtracting of Barrels, Firkins,
 and Gallons, there will arise some Difficulty by
 reason of the odd Half Gallons, or 5 Tenths, that
 are contained in a Firkin of Ale; which I shall
 endeavour to remove by the next Article, and to
 render the Business of Addition and Subtraction of
 those kind of Numbers as easy as if the Firkins
 consisted only of whole Gallons.

ART. III.

*To add and subtract Barrels, Firkins and
 Gallons, &c.*

I. Of Addition.

RULE.

IF the two given Numbers of Gallons and Tenths
 to be added together be under a Firkin, add
 them as in common Arithmetic: But if they are
 above a Firkin, (which will appear by Inspection)
 add the Thirds and Seconds together, (if there be
 any) as in common; and when you come to the
 Primes, always add 5 Tenths to the Sum of the
 Primes, and set down the Excess above 10 under-
 neath the Primes, and for 10 Primes carry 1 to the
 Gallons; then as many Gallons as there are above
 9 set down under the Gallons, and for the 9 Gal-
 lons carry 1 to the Firkins, and then proceed as
 usual.

EXAMPLE.

$$\begin{array}{r} \text{To} \quad \text{---} \quad 7.854 \\ \text{Add} \quad \text{---} \quad 3.645 \\ \hline 11.499 \end{array}$$

Here by Inspection I see that the Sum to be
 added will exceed a Firkin; therefore beginning
 at the Thirds Place, I say $5 + 4 = 9$, which I set
 underneath; then for the Seconds I say $4 + 5 = 9$, which

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9, which I set under the Seconds; then for the Primes I say $6 + 8 = 14$, and 5 more I borrow $= 19$. I set down the 9 under the Primes, and carry 1 to the Gallons; and $1 + 3 = 4$, and $7 = 11$, which is 2 above 9, and which I set down under the Gallons, and for the 9 Gallons I carry 1 to the Firkins; which is 1 F. 2 Ga. and .999 thousand Parts of a Gallon. This is so easy that it requires no more Examples.

II. Of Subtraction.

R U L E.

1. If the Minuend be greater than the Subtrahend, then subtract as usual. But if the Minuend be less than the Subtrahend (which will appear by Inspection) subtract the Thirds and Seconds as in common Arithmetic: But when you come to the Primes add 5 Tenths to the Primes of your Minuend, and subtract the Figure below from that augmented Prime, and set down the Remainder underneath the Primes; then add 8 to the Gallons of the Minuend, and subtract the Gallons from that Sum, and set down the Remainder underneath the Gallons; then carry 1 to the Firkins for the 8 Gallons you borrowed, and proceed as usual.

2. If after you have added 5 Tenths to the Primes you cannot have your Subtrahend Figure, then, instead of 5 Tenths, you must add 15 Tenths to it. But then always mind to carry 1 to the Gallons for the 15 Tenths borrowed. An Example or two will make this more intelligible.

E X A M P L E S.

	B. F. G.	B. F. G.
From —	0 : 1 : 2.999	1 : 0 : 1.3654
Take —	0 : 0 : 3.645	0 : 0 : 6.9832
Remainder	0 : 0 : 7.854	0 : 3 : 2.8822

In the first Example I say $9 - 5 = 4$, which I set down under the Thirds; and $9 - 4 = 5$, which I set down under the Seconds; then 6 from $9 + 5 = 14$ is 8, which I set down under the Primes; then 3 from $8 + 0 = 10$ is 7, which I set down under the Gallons: and for the 8 I borrowed I carry 1 to the Firkins; which taken from 1 leaves nothing, as above.

In the second Example I say $4 - 2 = 2$, which I set down under the Fourths, and $5 - 3 = 2$, which I set down under the Thirds, and 8 from $6 + 10 = 16$ leaves 8, which I set down under the Seconds. Then for the Primes I say $9 + 1 = 10$, from $3 + 15 = 18$, leaves 8, which I set down under the Primes, and carry 1 to the Gallons for the 15 borrowed, and say $1 + 6 = 7$, from

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 from $1 + 8 = 9$, leaves 2, which I set down under the Gallons, and carry 1 for the 8 Gallons I borrowed to the Firkins. The rest of the Work is performed as in common Arithmetic.

And in this Manner may any Number of Gallons and Firkins be added and subtracted with as much Ease and Expedition as if they were whole Numbers.

A R T. IV.

To gage and inch an Under Back whose Bases are equal Squares, or Parallelograms, and its Sides perpendicular to the Bases.

R U L E.

1. **W**ITH some convenient Instrument take the Length of the Back, which suppose here to be $= 148$ Inches, and also the Breadth $= 112$ Inches, and suppose the Depth $= 10$ Inches.

2. By Art. i. for a Square, or Art. ii. Chap. vii. for a Parallelogram, find the Area upon 1 Inch, which multiply by the Depth, and the Product will be equal to the Content of the whole Back.

3. To find the Content at every Inch of the Tun's Depth, reduce the Area of 1 Inch into Barrels, Firkins, and Gallons, which will be equal to the Content of the first Inch; and that Sum added to itself will be the Content of the second Inch; and to the Content of 2 Inches add the Content of the first Inch, and you will have the Content at 3 Inches deep; and thus, by continually adding the Content of the first Inch to the Contents of 3, 4, 5, 6, &c. Inches till the whole Depth is completed, you will have the Content of the Back at every Inch of its Depth. See the Work.

OPERATION.

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OPERATION.

112

112

206

148

148

282)16576(58.78 = Area in Gallons.

1410

10 = Depth.

2476

2256

587.80 = Content in Gallons.

. 2200

1974

B. F. G.

17 : 1 : 1.3 = Cont. reduced
to Barrels, &c.

. 2260

2256

Rem.—. . . 4

The Area of 1 Inch = 58.78 reduced to Bar-

B. F. G.

rels, Firkins and Gallons is $= 1 : 2 : 7.78$

1:2:7.78

Content at 2 Inches = 3 : 1 : 7.06

1:2:7.78

Content at 3 Inches = 5 : 0 : 6.34

I: 2: 7.78

Content at 4 Inches = 6 : 3 : 5.62

And thus, by continuing the Addition to 10 Inches = the Depth of the Back, I found the Contents at every Inch of the same as they are placed in the Table in Page 115, which is so easy, that it is quite needless to insert the whole Process. Only here observe that if the Content of the last Inch is equal to the Content of the Back before found, the Work is done right, otherwise not.

A R T. V.

To gage a Back or Cooler, and to find the Contents at every Tenths of an Inch, commonly called tenting a Back.

RULE

1. **WITH** some convenient Instrument take the Length and Breadth of the Back.

2. By Art. x. Chap. vii. I find the Area or Content on one Lock deep, which reduce to Barrels, Fiskins.

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 Fathoms, and Gallons; find the one Tenth of that Area by removing the Dot one Place further to the Left Hand, which also reduce to Barrels, &c.

3. To find the Content at every Tenth, add the Area or Content of the first Tenth to itself, and you will have the Content at 2 Tenths; again, add the Content of the first Tenth to the Content of the second Tenth, and you will have the Content of three Tenths; and thus, by continually adding the Content of the first Tenth for every Tenth, you may tenth a Back to what Depth you please. But if you continue the Table to 5 or 6 Inches of the Back's Depth it will be sufficient in most Cases, because the Worms are seldom put deeper in them than 6 Inches for cooling; and if the Gage taken should exceed the Limits of the Table, it is only taking the Depth at twice out of the Tables, and adding the Contents of each Depth together, and you will have the Content at the whole Depth of the Gage.

EXAMPLE.

Suppose I found the Length of the Back = 372 Inches of the Breadth = 90.8 Inches: To find the Content at every Tenth of the first 3 Inches of the Back's Depth.

OPERATION.

Length = 372

Breadth = 90.8

2176

24480

282)24697,6(87.58 = Area on 1 Inch in Gallons.

2356

2137

B. F. G.

1974

3 : 2 : 2,58 = Area 1 Inch reduced.

21636

1410

2260

2356

Rem. — . . . 4

The Area or Content of 1 Inch being 87.58 Gallons, the tenth Part of that Sum is 8.758 Gallons, which reduced is O.B. 1 F. 2.58 G. and so much will the Back hold on every Tenth of an Inch of its Depth. The next Thing to be done is to find the Content at every Tenth of an Inch till you come to the Depth required to be infused in the Table thus.

OPERATION.

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OPERATION.

B. F. G.

$$\begin{array}{r} 1 \text{ Tenth} = 0:1:0.358 \\ 0:1:0.358 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \text{ Tenth} = 0:2:0.516 \\ 0:1:0.258 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \text{ Tenth} = 0:3:0.774 \\ 0:1:0.258 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \text{ Tenth} = 1:0:1.032 \\ 0:1:0.258 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \text{ Tenth} = 1:1:1.290 \\ 0:1:0.258 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \text{ Tenth} = 1:2:1.548 \\ 0:1:0.258 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \text{ Tenth} = 1:3:1.806 \\ 0:1:0.258 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \text{ Tenth} = 2:0:2.064 \\ 0:1:0.258 \\ \hline \end{array}$$

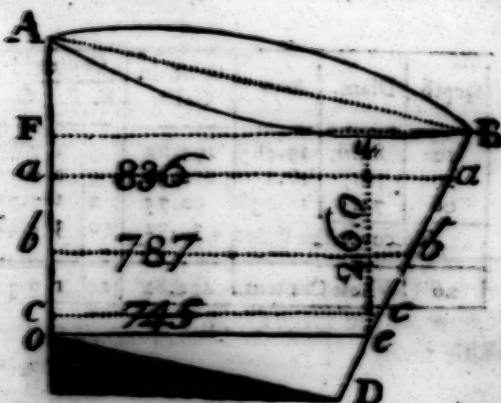
$$\begin{array}{r} 9 \text{ Tenth} = 2:1:2.322 \\ 0:1:0.258 \\ \hline \end{array}$$

$$\text{Cont. at 1 Inch} = 2:2:2.580$$

Thus when I come to 1 Inch of the Back's Depth, I find the Content to be equal to that before found, which proves the Work to be true; and in this Manner I proceeded to make the Table in Page 116 to 3 Inches of the Back's Depth, where all the above Contents are placed to the nearest even Gallon.

A R T. VI.

To gage and inch a Gaile Tin in Form of the Frustrum of a Cone, and to make an Allowance for the Drip or Fall. See the Figure following.



RULE.

R U L E.

1. **C**AUSE Water to be poured into the Tun till the Bottom is just covered, which will be when the Water touches the Point *e*, and suppose the Measure of the Water in this Case to be 30.92 Gallons.

2. With a Quadrant, or a Level, find the horizontal Line *r a* thus: Fasten a String on the Top Edge of the Tun, as at *a*, and let an Assistant hold it out on the other Side. Then rest your Level on the same Place, and hold it in such a Manner as that the Bottom of it may be horizontal, which you will find by the Plummets. Then let your Assistant draw out the String parallel to the Bottom of the Level, till it touches the opposite Side at *r*, where fasten it; then will *r a* represent the Surface of the Liquor when the Tun in that Position is full, and it will be also parallel to the Line *oe*, or the Surface of the Liquor which covers the Drip.

3. Take the perpendicular Depth of the Tun, that is, the shortest Distance from the Water at *e* to the String at *a*, which suppose here to be = 26 Inches = *o f* = *g e*.

4. Take Mean Diameters parallel to *r a* and *oe* in the Middle between every 6 or 10 Inches of that Depth; and suppose the first Mean Diameter *aa* at 5 Inches from *r a* = 83.6 Inches; the second Mean Diameter *bb* at 15 Inches from *r a* = 78.7 Inches; and the third Mean Diameter *cc* at 25 Inches from *r a* = 74.5 Inches.

5. By the Table of Ale Areas in Chap. xv. find the Areas and set them against their respective Diameters, as in the third Column of the following Table. Then find the Contents of the several Depths in Gallons, and set them in the fourth Column. Lastly, convert these Contents into Barrels, &c. and place them as in the three last Columns of the Table.

Depth.	Diam.	Area.	Cont. in Galls.	Content in		
				B.	F.	G.
10	83.6	29.465	194.65	5	2	7.65
10	78.7	27.350	172.50	5	0	2.50
06	74.5	15.458	92.75	2	2	7.75
To cover the Drip.			30.92	0	3	5.42
26	Whole Content.		490.82	10	2	6.32

Now

Now to find the Content at every Inch of the Depth reduce the first Area 19.465 into Barrels, &c. and it will be 0 B. 2 F. 2.465 G. which subtracted from the whole Content = 14 B. 1 F. 6.32 G. leaves 13 B. 3 F. 3.855 G. = the Content when 1 Inch is dry. From that Remainder subtract the said Area, and you will have the Content when 2 Inches are dry; and thus continue subtracting the first Area from the several Remainders until 10 Inches are dry, when there will remain 8 B. 2 F. 7.17 Gs. Then take the second Area 17.25 converted into Firkins, &c. and subtract it from the last Remainder, and thus continue to do till you have 20 Inches dry, when there will remain 3 B. 2 F. 4.67 Gs.

Lastly, take the third Area 15.458 converted into Barrels, &c. and subtract it from the Remainder when 20 Inches are dry; and thus proceed till you have 26 Inches dry, when you will have remain, if no Mistake be made, 0 B. 3 F. 5.42 G. = the Quantity of Liquor to cover the Bottom, as before found.

I have omitted to shew the Operation at large, because it is performed just in the same Manner as the Copper in Art. ii. of this Chapter. But the Contents at every dry Inch in Barrels are inserted in the Table Page 116, to the nearest even Gallon.

N.B. You must make a Mark at r for a constant Dipping Place while the Tun is in this Position.

If this Tun had been a Cylinder, or a Square, whose Bases are equal and parallel, and whose Sides are perpendicular to their Bases, then it is plain that the dry Hoof $r \Delta n$ would be equal to the wet Hoof $e n$, and the Solidity of the Hoof $e n$ might easily be found either by Measure or Calculation; for if half the Depth of the Water at $e n$ be multiplied by the Area of the Base, the Product will be the Content of the Drip, which subtracted from the whole Content of the Tun, will leave the Content of it in that Position, and the Dipping Place will be the same Distance from Δ as e is from n , that is, Δr must be $= e n$. In like Manner may Tuns of any other Form be gaged and inched, provided the Areas are found at the several Depths according to the Directions delivered in Chap. vii.

Now having fully explained the Nature of gaging and inching, &c. of C. Brewers Utensils, I shall, in the next Place, give a Precedent of a Table Book, which was collected from the foregoing Operations.

A Table Book.

A— B—, Common Brewer.

No. and Names.	Depth.	Diameter.	Length.	Breadth.	Area.	Content in Gallons.	Content in		
							B. F.	G.	Q. B. G.
MT 30.0	—	119	102	53.47	534.7	—	—	25	
10	88.0	—	—	21.568	215.680	6	13.180	—	
10	85.5	—	—	20.360	203.600	5	38.10	—	
07	82.5	—	—	18.956	182.692	3	35.192	—	
03	To cover the Crown.					28.044	0	32.544	
30	The whole Content.					580.016	17	02.016	
UB 10	—	118	112	58.78	587.80	17	1	1.3	
Back —	—	272	90.8	87.58	—	2	2	2.58	
10	83.6	—	—	19.665	194.65	5	2	7.65	
10	77.7	—	—	17.250	172.50	5	0	2.50	
05	74.5	—	—	15.458	92.75	2	0	7.75	
To cover the Bottom.					30.92	0	3	5.42	
The whole Content.					490.82	14	1	6.32	

The above Table contains the Dimensions, Areas, and Contents of the several Utensils, as they were found in the several Articles of this Chapter, and which are supposed to be all that are made use of by a Common Brewer. For if there were any other Utensils used by the Brewer they should all be gaged and inched according to the foregoing Directions, and their Dimensions, Areas, and Contents be inserted in the above Table. As for Instance, if there were two Backs, or two Rounds, or two Squares, write in the first Column 1 Back and 2^d Back, or 1 Round and 2^d Round, or 1 Square and 2^d Square; and then their Dimensions, &c. against them. In the two following Pages are placed the Contents at every Inch, &c. of the Depth of each Utensil; their Use is for casting up of Common Brewers Gages, or finding the Content at any Inch of the Depth. Here it will not be amiss to observe that the wet Inches are taken in the Mash Tun, Under Back, and Backs; but the dry Inches in Coppers, Rounds, and Squares.

A Common Brewer's Table Book.

Mash Tun.				Copper.				Under Back.			
Wet Inches.	Q.	B.	G.	Dry Inch	B.	F.	G.	Wet Inch	B.	F.	G.
1		6	5	Full	1	0	2	1	1	1	8
2		1	3		2	1	3	2	3	1	7
3		2	4		3	2	4	3	5	1	6
4		3	5		4	3	5	4	6	1	5
5		4	6		5	4	6	5	8	1	4
6		5	7		6	5	7	6	10	1	3
7		6	8		7	6	8	7	11	1	2
8		7	9		8	7	9	8	13	1	1
9		8	10		9	8	10	9	15	1	0
10		9	11		10	9	11	10	1	1	1
11	9	1	4		11	10	12	Half Inch	0	3	4
12	10	2	5		12	11	13				
13	11	3	6		13	12	14				
14	12	4	7		14	13	15				
15	13	5	8		15	14	16				
16	14	6	9		16	15	17				
17	15	7	10		17	16	18				
18	16	8	11		18	17	19				
19	17	9	12		19	18	20				
20	18	10	13		20	19	21				
21	19	11	14		21	20	22				
22	20	12	15		22	21	23				
23	21	13	16		23	22	24				
24	22	14	17		24	23	25				
25	23	15	18		25	24	26				
26	24	16	19		26	25	27				
27	25	17	20		27	26	28				
28	26	18	21		28	27	29				
29	27	19	22		29	28	30				
30	28	20	23		30	29					
				To cover the Cu.							
				1 half In.	0	1	2.5				
				2 half In.	0	1	2				
				3 half In.	0	1	1.5				

Of the Use of the Tables.

EXAMPLE I.

Suppose the Depth of the Goods or Grains were 18 Inches in the Mash Tun, how many Quarters &c. would it contain at that Depth?

I seek for 18 in the first Column under wet Inches, and against it is 15 Qrs. 0 Bs. 3 Ga. = the Content.

Note, In gaging you must only take even Inches in the Mash Tun. But the Copper, Under Back, Rounds, and Squares, must be taken to the nearest Half Inch, and the Backs to the nearest Tenth.

A Common Brewer's Table Book.

Sack.				Round.			
Wet Inches and Tenths.	B.	F.	G.	Dry Inch	B.	F.	G.
.1	0	1	0	Full.	14	16	
.2	0	2	1	1	13	14	
.3	0	3	1	2	13	13	
.4	1	0	1	3	12	12	
.5	1	1	1	4	12	11	
.6	1	2	2	5	11	11	
.7	1	3	2	6	11	10	
.8	2	0	2	7	10	10	
.9	2	1	2	8	9	9	
1.0	2	2	3	9	9	8	
.1	2	3	3	10	8	8	
.2	3	0	3	11	8	7	
.3	3	1	3	12	7	7	
.4	3	2	4	13	7	6	
.5	3	3	4	14	6	6	
.6	4	0	4	15	6	5	
.7	4	1	4	16	5	5	
.8	4	2	5	17	5	4	
.9	4	3	5	18	4	4	
1.0	5	0	5	19	4	3	
.1	5	1	5	20	3	3	
.2	5	2	6	21	3	2	
.3	5	3	6	22	2	2	
.4	6	0	6	23	2	1	
.5	6	1	6	24	1	1	
.6	6	2	7	25	1	0	
.7	6	3	7	26	0	0	
.8	7	0	7	Drip	—	—	
.9	7	1	7	1 half Inch	0	11	
1.0	7	2	8	2 half Inch	0	10	
				3 half Inch	0	9.5	

EXAMPLE II.

Suppose there were 15 Inches of the Copper dry, how many Barrels, &c. would there be in it?

OPERATION.

I seek for 15 in the first Column of the Copper under dry Inches, and I find 7 Ba. 2 Fa. 8 Ga. = the Content. When the Depth is taken to a Half Inch, then the Contents of the Half Inches under the Tables will be of Use. But observe when the wet Inches are taken, then the Content of the Half Inch must be added; and when the dry Inches are taken, the Content of the Half Inch must be subtracted from the Content of the even Inches.

As for Example, suppose 15 $\frac{1}{2}$ Inches of the Copper were dry, how much would it then contain? The Content at 15 Inches, as found above, is 7 Ba. 2 Fa. 8 Ga. from which I subtract 5 B. 1 F. 2 G. the Area of the 2^d half Inch, and there remain 7 Ba. 1 F. 6 Ga. = the Content required.



C H A P. XI.

Of Cask Gaging.

CASK Gaging is a very curious Art, and is the most difficult Part of Gaging.

The Difficulty arises from the great Variety of Curves that these Kind of Vessels are composed of, and for that Reason it requires a good deal of Judgment, and Practice, to determine the exact Content of any close Vessel. In order to have a right Understanding in the Matter, it would be necessary for the Gager to have some Idea of the Conic Sections, because Casks are generally compared to Solids generated by one or other of those Sections; which would enable him the better to distinguish to which of these Solids the Cask's Curve bears the nearest Resemblance.

Gagers have reduced those different Curvatures of Casks to four Varieties, &c.

Var. 1. Those Casks that are most curved, or bent, are considered as the middle Frustrum of a Spheroid, which is represented by the outer Lines of the following Figure, viz. $AIOIBCI\alpha IN$.

Var. 2. When the Staves are not so much arching, or bent, the Cask is supposed to be the middle Frustrum of a Parabolic Spindle, which is represented by the Lines $AIOIBCIC\alpha IN$.

Var. 3. When the Staves are but little bent, or curved, they are reputed to be in the Form of the Frustrums of two Parabolic Conoids joined together at their greatest Bases, and is represented by the Lines $AIOIBCIC\alpha IN$.

Var. 4. When the Lines are strait from Head to Bung, so as at the Lines $AIOIBCIC\alpha IN$, then it is plain that such Cask is formed of the Frustrums of two Cones joined together at their greater Bases.

In the Practice of Cask gaging the Officers of the Excise are obliged to follow one general Method, wherein they suppose that a Cask may have the same Dimensions with respect to its Head, Bung, and Length, and yet by reason of the great or little Archness of the Cask's Staves between the Head and Bung, the Cask may belong to either

of the four Varieties; which is according to Mr. Edwards's System of Cask gaging, and agreeable thereto are the several Varieties graduated on the Sliding Rule, in order to reduce each Variety to a Mean Diameter, or that of a Cylinder whose Height and Capacity is nearly equal to that of the Cask.

And as the Contents of Casks are generally found by the Rule, and that being the Test whereby all those Contents must be tried, I shall all along confine myself to such Rules as will produce the same Content as tho' they were cast by the Sliding Rule; so the young Gager may use which Method he thinks most proper.

A R T. I.

To find the Content of a Cask in any of the four Varieties both by Pen and Rule.

R U L E.

1. To find a Mean Diameter.

THE Casks being reduced to four Varieties, if you multiply the Difference between the Head and Bung Diameters when it is less than 6 Inches,

By $\left\{ \begin{array}{l} .68 \\ .62 \\ .55 \\ .5 \end{array} \right\}$ for the $\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array} \right\}$ Variety.

Or if the Difference between the Head and Bung exceed 6 Inches,

By $\left\{ \begin{array}{l} .7 \\ .64 \\ .57 \\ .52 \end{array} \right\}$ for the $\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array} \right\}$ Variety.

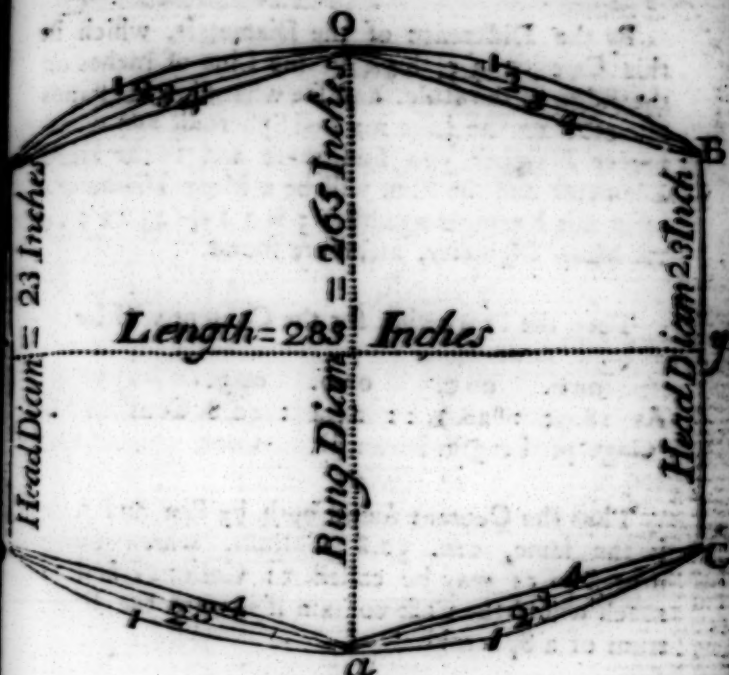
and add the Product to the Head Diameter, then that Sum will be a Mean Diameter.

2. To find the Content.

Square the Mean Diameter, and multiply that Square by the Length of the Cask, and the Product multiply or divide by the Factors, &c. as Page 49, and the Product or Quotient will be equal to the Content of the Cask.

By Art. in Chap. vii. find the Area of the Circle, which Area multiplied by the Length will be equal to the Content of the Cask.

The Figure of the four Varieties of Casks.



EXAMPLE.

Let the above Figure represent all the 4 Varieties of Casks, whose Bung Diameter oa is $= 26.5$ Inches, Head Diameter $AD = BC = 23$ Inches, and Length $xy = 28.3$ Inches: To find the Content in Ale Gallons, supposing it to belong to all the four Varieties

1. For the Spheroid, or 1st Variety.

Operation by the Pen.

Bung D. $= 26.5$ M. D. $\left. \begin{array}{l} \\ \end{array} \right\} = 25.4$
Head D. $= 23.0$ nearly

Diff. $= 3.5$

Factor $= .68$

280

210

Prod. $= 33.80$ Sq. of M. D. $\left. \begin{array}{l} \\ \end{array} \right\} 645.16$

Head D. $= 23$ Length of Cask $\left. \begin{array}{l} \\ \end{array} \right\} 28.3$

Mn. D. $= 25.38$

103548

516128

129832

359, 85, 18, 5, 8, 0, 2, 8 (50.8 = Cask

17951

103548

129832

103548

129832

103548

129832

103548

129832

By the Rule.

By the Difference of the Diameters, which in this Case = 3.5, look on the Line of Inches on the Edge of the Rule, and see what Number stands against it on the Line marked Spheroid, and whatsoever Number you find there add to the Head Diameter and the Sum will be a Mean Diameter; as in this Example against 3.5 is $24 + 23 = 25.4$ = Mean Diameter, as before found.

Then the Proportion for the Content will be

OND.	ONC.	OND.	ONC.
As 18.95 :	28.3 ::	25.4 :	50.8 Cont. in Ale
Gage-pt. Length.	Mean. D.	Cont.	Gi.

Thus the Content found both by Pen and Rule is the same, viz. 50.8 Gallons, which being nearest to 51 may be called 51 Gallons; and so much would the Cask contain if it were the Frustrum of a Spheroid.

2. For the Middle Frustrum of a Parabolic Spindle, or that of the 2d Variety.

Operation by Pen.

Diff. of } = 3.5	Mean D. } = 25.2
Diams. } nearly	
Factor. = 62	= 25.2

70	504
210	1260
	504

Prod. = 2170	
H. D. = 23.	Sq. of } = 633.04
	M. D. }
M. D. = 25.170	Length of Cask } = 28.3

100312
508032
127008

359.05	17971.632	50.0 = Cont.
	179525	in Ale Ga.

Rem. .. 19132

By the Rule.

With the Difference of the Diameters look on the Line of Inches on the Edge of the Rule, and whatsoever Number stands against it on the Line marked 2d Variety add to the Head Diameter and the Sum will be a Mean Diameter.

In this Case the Difference is 3.5, against which

Of Cyl. Gages.

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on the Line marked 2d Variety is 2.3 nearly, which added to the Head Diameter = 25.3 = the Mean Diameter before found.

Then the Proportion for the Content will be

on d. on c. on d. on c.

As 18.95 : 28.3 :: 25.3 : 50 = as before
Gage-pt. Length. Mean D. Content. found.

Thus the Content found both by Pen and Rule is the same, viz. 50 Gallons, and so much would the Cask contain if it were in the Form of a Parabolic Spindle called the 2d Variety.

3. For the Frustrum of two Parabolic Conoids, or 3d Variety of Casks.

Operation by Pen.

Diff. of } = 3.5 Mean D. } = 24.9
Dms. } nearly

Factor = .55 24.9

475 2241

175 996

498

Prod. = 1.925

H.D. = 23. Sq. of } = 620.01
 M. D. }

M. D. = 24.925 Length = 28.3

186003

496008

124002

359.0517546, 283(48.8 = Cont.

143620

in Alc Ga.

.312428

287240

.311883

287240

Remains .24643

By the Rule.

With the Difference of Diameters look on the Line of Inches on the Edge of the Rule, and the former Number standing against it on the Line marked 3d Variety add on the Head Diameter, and the Sum will be a Mean Diameter.

In this Case the Number standing against 1.5 on the Line marked 3d Variety is 1.9 + 23 = 24.9 = Mean Diameter, as before found. Then the Proportion for the Content is

on d. on c. on d. on c.
As 18.95 : 28.3 :: 25.3 : 50 = as before
Gage-pt. Length. Mean D. Content. found.

The Content found both by Pen and Rule agree very well, for they both may be called 49 Gallons, that being the nearest even Gallon; and so much would the Cask contain if it were in the Form of the Frustrum of two Parabolic Conoids, or 3d Variety.

4. For the Frustrum of two Cones joined together at their greater Bases, called the 4th Variety.

Operation by Pen.

$$\begin{array}{r} \text{Diff. of } \left. \begin{array}{l} \text{Diams.} \\ \text{Factor} \end{array} \right\} = 3.5 \\ \text{Factor} = .5 \end{array}$$

$$\begin{array}{r} \text{Prod.} = 1.75 \\ \text{H. D.} = 23. \end{array}$$

$$\text{M. D.} = 24.75$$

$$\text{Square of Mean Diam.} = 612.5625$$

$$\begin{array}{r} 24.75 \\ \times 24.75 \\ \hline 12375 \\ 17325 \\ 9900 \\ 4950 \\ \hline 6125625 \end{array}$$

$$\begin{array}{r} 18376875 \\ 49005000 \\ 12251250 \\ \hline 359051875 \end{array}$$

$$359.051875 \div 48.281 = \text{Con. in Ale Ga.}$$

$$\begin{array}{r} 297351 \\ 287240 \\ \hline 101118 \end{array}$$

$$\begin{array}{r} 101118 \\ 71810 \\ \hline 293087 \end{array}$$

$$\begin{array}{r} 293087 \\ 287240 \\ \hline 58475 \end{array}$$

$$\begin{array}{r} 58475 \\ 35905 \\ \hline 22570 \end{array}$$

Remains — 22570

By the Rule.

With the Difference of Diameters look on the Line of Inches on the other Edge of the Rule, and whatsoever Number stands against it on the Line marked F C add to the Head Diameter, whose Sum will be a Mean Diameter.

In this Case the Number standing against 3.5 on the Line F C is 1.77 nearly, which added to the Head Diameter = 24.77, the Mean Diameter.

Then for the Content the Proportion is

$$\begin{array}{r} \text{Mean Diam.} : \text{Head Diam.} :: \text{Head Diam.}^3 : \text{Content} \\ 24.77 : 23 :: 23^3 : \text{Content} \end{array}$$

The Content found both by Pen and Rule is nearly alike, and may be called 48, because it is the nearest even Gallon.

In like Manner might the Content of the several Varieties have been found in Wine Measure, if instead of dividing the Product of the Square of the Mean Diameter, and the Length of the Cask by the Ale Divisor = 359.05, I had made Use of 294.12, the Wine Divisor.

And the Proportion by the Rule for Wine Gallons would be, As the Wine Gage-point on D is to the Length of the Cask on C, so is the Mean Diameter on D to the Content in Wine Gallons on C.

As for Example, If the Content of the 1st Variety was required in Wine Gallons, the Proportion is

on D. on C. on D. on C.
As 17.15 : 28.3 :: 25.2 : 61.1 = Wine Ga.
W. Gage- Length. M. Diam. Content.
point.

And in this Manner may the Content of the other three Varieties be found in Wine Gallons.

ART. II.

To find the Content of a Cask supposed to belong to all the four Varieties, when the Difference of the Head and Bung Diameter is above 6 Inches.

EXAMPLE.

LET the Cask's Head Diameter = 24.5 Inches, Bung Diameter = 31.5 Inches, and the Length = 42 Inches: To find the Content in Ale Gallons in all the four Varieties.

OPERATION.

Bung Diam. = 31.5

Head Diam. = 24.5

Diff. Diam. = 7.0

Head Diam. = 24.5

$$\left. \begin{array}{l} 7 \times .7 = 4.9 \\ 7 \times .61 = 4.27 \\ 7 \times .57 = 3.99 \\ 7 \times .52 = 3.64 \end{array} \right\} \begin{array}{l} M. \\ D. \\ A. \\ W. \end{array}$$

The A. = 2.407

W. = 2.379

the M. = 2.601

the D. = 2.407

which x 42 = 101.294

by 42, the

length of

the Cask = 4268.46

A R T. III.

To find the Content of all the Varieties of Casks another Way.

E X A M P L E.

LET the Cask be supposed to have the same Dimensions as that in the last Article: To find the Content in Ale Gallons in all the four Varieties.

1. For the Spheroid, or 1st Variety.

R U L E.

To twice the Square of the Bung Diameter add the Square of the Head Diameter, and multiply this Sum by the Length of the Cask, and the Product divide by 1077 ($= 3 \times 359$, the Circular Divisor for Ale Gallons in Page 49) and the Quotient will be $=$ to the Content in Ale Gallons.

O P E R A T I O N.

Twice the Square of the } = 1984.5
 Bung Diam. 31.5 — — }
 Square of Head Diam. 24.5 = 600.25
 Sum ————— = 2584.75
 Which multiply by the Lnth. = 42

1077)108559.5(100.8 =
 Content.

If the Product of the Sum of the Squares and Length of the Cask were divided by 883 $= 3$ Times the Circular Divisor for Wine Gallons in Page 49, the Quotient would have been $=$ the Content in Wine Gallons.

2. For the 2d Variety of Casks.

To the Sum and half Sum of the Squares of the Head and Bung Diameters add three Tenth Parts of the Difference of the said Squares, and this Sum multiply by the Length of the Cask and that Product divide by 1077, and the Quotient will be $=$ to the Content of the Cask in Ale Gallons.

O P E R A T I O N.

Square of B. D. 31.5 = 992.25 Diff. Sq. = 392.
 Square of H. D. 24.5 = 600.25 $\frac{1}{10}$ of Diff. = 39.2
 Sum of Squares = 1592.50
 $\frac{1}{2}$ Sum of Squares = 796.25 $\frac{1}{10}$ of Diff. = 39.2
 $\frac{3}{10}$ of the Diff. of Sq. = 117.6
 Sum ————— = 2506.35
 Which x by Length = 42

1077)105266.7(97.7 = Cont. in Ale
 Galls.

3. For the third Variety of Casks,

R U L E.

To the Sum and half Sum of the Squares of the Head and Bung Diameters add one Tenth of the Difference of their Squares, and this Sum multiply by the Length of the Cask, and that Product divide by 3 Times the Circular Divisor in Page 49; the Quotient will be equal to the Content of the Cask of the same Denomination as the Divisor made use of.

O P E R A T I O N.

$$\begin{array}{rcl}
 \text{Square of B. D. } 31.5 & = & 992.25 \\
 \text{Sq. of H. D. } 24.5 & = & 600.25 \\
 \hline
 \text{Sum of Squares} & = & 1592.50 \\
 \frac{1}{2} \text{ Sum of Squares} & = & 796.25 \\
 \frac{1}{10} \text{ of Diff. of Sq.} & = & 39.2 \\
 \hline
 \text{Sum} & = & 2437.95 \\
 \text{Which } \times \text{ by Length } 42 & = & 102493.9 \\
 \hline
 1077 \overline{) 102493.9} & = & 94.7 = \text{Cont. in Alc Ga.}
 \end{array}$$

4. For the 4th Variety of Casks,

R U L E.

From the Sum and half Sum of the Squares of the Head and Bung Diameters subtract half the Square of the Difference of the Diameter; then multiply the Remainder by the Length of the Cask, and the Product divide by three Times the Circular Divisor in Page 49; the Quotient will be equal to the Content of the Cask of the same Denomination as the Divisor made use of.

O P E R A T I O N.

$$\begin{array}{rcl}
 \text{Square of B. D. } 31.5 & = & 992.25 \\
 \text{Sq. of H. D. } 24.5 & = & 600.25 \\
 \hline
 \text{Sum of Squares} & = & 1592.50 \\
 \frac{1}{2} \text{ Sum of Squares} & = & 796.25 \\
 \hline
 \text{Sum} & = & 2388.75 \\
 \text{Sq. of Diff. } 49 & = & 24.5 \\
 \frac{1}{2} \text{ Sq. of Diff. } 24.5 & = & 12.25 \\
 \hline
 \text{Remainder} & = & 2364.25 \\
 \text{Which } \times \text{ by Length } 42 & = & 99298.5 \\
 \hline
 1077 \overline{) 99298.5} & = & 92.2 = \text{Content in Alc Ga.}
 \end{array}$$

The several Contents thus found agree very well with those found by the former Method in the last Article; so either Rule may serve for the finding of the Content of a Cask belonging to any of the four Varieties by the Pen. But the Sliding Rule is preferable to either, because by that the Contents are found with much more Ease and Expedition, and near enough the Truth in Practice.

A R T. IV.

To find the Content of a Cask in the Form of a Spheroid by the Lines c and d on the Rule, without the Help of the Lines of Varieties.

R U L E.

SET the Length of the Cask on c

to $\left\{ \begin{array}{l} 32.8 \\ 29.7 \end{array} \right\}$ for $\left\{ \begin{array}{l} \text{Ale} \\ \text{Wine} \end{array} \right\}$ Gallons on d.

Then seek for the Bung Diameter on d, and note the Number against it on c, which double; then seek the Head Diameter on d, and note the Number against it on c, which Number added to double the other before found will = the Content of the Cask in Ale or Wine Gallons respectively.

E X A M P L E.

Let the Dimensions of the Cask be the same as that in the two last Articles, viz. Head Diameter 24.5, Bung Diameter 31.5, and the Length 42 Inches. To find the Content both in Ale and Wine Gallons.

1. For the Content in Ale Gallons.

Set 42 = the Length on c to 32.8 on d, then against the Bung Diameter = 31.5 on d is 38.7 on c, which doubled = 77.4; then seek for 24.5 = the Head Diameter on d, and against it is 23.5 on c, and $77.4 + 23.5 = 100.9$ = the Content of the Cask in Ale Gallons, which is very near the same as found by the former Rules.

2. For the Content in Wine Gallons.

Set 42 = the Length on c to 29.7 on d, then against the Bung Diameter 31.5 on d is 47.2 on c, which doubled = 94.4; next find the Head Diameter 24.5 on d, and against it is 28.7 on c, and $94.4 + 28.7 = 123.1$ = the Content in Wine Gallons.

This is a very useful Rule for finding the Contents of the 1st Variety of Casks, because it may be performed in less Time this Way than by the 4th Varieties on the Rule.

ART.

A R T. V.

To find the Content of a Cask without regarding the Variety at all, by having a fourth Dimension given that is a Diameter in the Middle between the Head and Bung of the Cask.

R U L E.

SET the Length of the Cask on c

to $\left\{ \begin{array}{l} 46.4 \\ 42.0 \end{array} \right\}$ for $\left\{ \begin{array}{l} \text{Ale} \\ \text{Wine} \end{array} \right\}$ Gallons on d,

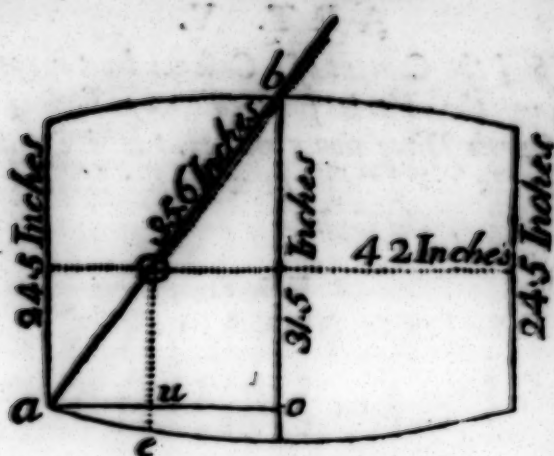
Then find the Head, Bung, and Middle Diameters on d, and note the Numbers against them on c; then, if to the Sum of the first and second of these Numbers there be four times the third added, that Sum will be equal to the Content of the Cask in Ale or Wine Gallons. There is one Difficulty arising from this Rule, viz. the finding of a Diameter in the Middle between the Head and Bung of the Cask. But this may be effected by a Plumb Line, according to the following Directions.

1. Get a Ring just to fit your Gauging Rod, so as to slide up and down at pleasure; file a small Notch in the Inside of the Ring for the Line to move freely up and down with the Weight of the Plumbet.

2. Put your Gauging Rod into the Bung Hole of the Cask, and measure the Diagonal ba , (see the following Figure) which I suppose $= 35.6$ Inches; then take out the Rod again, and fix the Ring on it at 17.8, equal to Half the Diagonal, as at $\odot$, and put the Gauging Rod again into the Cask, with the Plumb Line running through the Notch of the Ring till it just touch the Side of the Cask at e ; then holding the Thread close to the Rod take it out of the Cask and measure the Distance $\odot e$, that is the Length of the Line from the Point of Suspension to the Extremity of the Plumbet, which I suppose $= 16.7$. This reserve.

3. From Half the Bung Diameter subtract one fourth Part of the Difference between the Head and Bung Diameters. The Remainder subtract from the reserved Number, and this last Remainder double and add to the Head Diameter, and the Sum will be equal to the middle Diameter required.

* The Triangles abc and $a\odot e$ being similar, therefore when $b\odot$ is half of ab , ae will be half of ac . Let the Bung Diameter $= b$, the Head Diameter $= a$, $\frac{1}{2}$ the Length $= L$, and the Diagonal $= x$; then $L : b - \frac{1}{2}x :: \frac{1}{2}L : a - \frac{1}{2}x = \odot a$, and $\odot e - \odot a = ae$, and $2ae + a =$ the middle Diameter. EXAM-



EXAMPLE.

Suppose the Head Diameter of the Cask above = 24.5 Inches, Bung Diameter = 31.5, and Length = 42 Inches: To find the Content in Ale Gallons.

OPERATION.

$\frac{1}{2}$ Bung Diam 31.5	= 15.75	R. Diam. = 31.5
$\frac{1}{2}$ of Diff. of Diam.	= 1.75	H. Diam. = 24.5
1st Remainder	= 14.00	Diff. = 7.0
Length $\odot$ found	= 16.7	$\frac{1}{2}$ Diff. = 1.75
2d Remainder	= 2.7	
Double the last Rem.	= 5.4	
Head Diam.	= 24.5	
Middle Diam.	= 29.9	

Then for the Content.

Set 42 on c to 46.4 on d, then against the Head Diameter 24.5 on d is 11.73 on c; against the Bung Diameter 31.5 is 19.4; and against the middle Diameter 29.9 is 17.44, which multiplied by 4 = 69.76, and $11.73 + 19.4 + 69.76 = 100.89 =$ the Content of the Cask in Ale Gallons.

If, instead of setting the Length of the Cask to 46.4, I had set it to 42, then the Content would have been found in Wine Gallons.

This Rule is universal for all Sorts of tapered Casks, let their Form or Variety be of any kind whatsoever. See Mr. Shirdif's Gauging, Page 200, &c.

A R T. VI.

To find the Content of a spheroidal Cask, by having the Length of the Diagonal Line given, without knowing the Dimensions of the Head, Bung, and Length at all.

1. By the Branan's Rule.

PUT the Branan's Rule into the Bung Hole of the Cask (supposing it to be in the Middle of the Cask) till the End of it meet the Intersection of the Head, and the Staves on the opposite Side; then on that Part of the Rule held in the Center of the Bung Hole on the Diagonal Lines is the Content in Ale and Wine Gallons. This is so easy that it requires no Example.

2. By the Lines *d* and *z* on the Rule.

Set 30 on *d*

to $\left\{ \begin{matrix} 60 \\ 73.3 \end{matrix} \right\}$ for $\left\{ \begin{matrix} \text{Ale} \\ \text{Wine} \end{matrix} \right\}$ Gallons on *z*:

Then against any Diagonal on *d* is the Content upon *z* in Ale or Wine Gallons respectively.

E X A M P L E.

Suppose the Length of the Diagonal of a Cask = 28.8 Inches, what will the Content be both in Ale and Wine Gallons?

1. Proportion for Ale Gallons.

on *d*. on *z*. on *d*. on *z*.
As 30 : 60 :: 28.8 : 53. = Ale Gallons.
Diag. Content. Diag. Content.

2. Proportion for Wine Gallons.

on *d*. on *z*. on *d*.
As 30. : 73.3 :: 28.8 : 65 = Wine Galla.
Diag. Content. Diag. Content.

3. By the Lines *d* and *c* on the Rule.

Set the Length of the given Diagonal of the Cask

on *c*

to $\left\{ \begin{matrix} 21.2 \\ 19.2 \end{matrix} \right\}$ for $\left\{ \begin{matrix} \text{Ale} \\ \text{Wine} \end{matrix} \right\}$ Gallons on *d*.

Then against that Diagonal on *d* is the Content upon *c* in Ale or Wine Gallons respectively.

E X A M P L E.

Let the Diagonal = 28.8 Inches as before: To find the Content of the Cask in Ale and Wine Gallons.

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Proportion for Ale Gallons.

on D. on C. on D. on C.
As 21.2 : 28.8 :: 28.8 : 53. = in Ale Gallons.
Gage-pt. Diag. Diag. Content.

2. Proportion for Wine Gallons.

on D. on C. on D. on C.
As 19.2 : 28.8 :: 28.8 : 65 = Wine Galls.
Gage-pt. Diag. Diag. Content.

Thus the Contents found either Way are the same; as also by the Branan's Rule, for again 28.8 on the Line of Inches there is placed

53. Ale } Gallons,
65 Wine }

on the Diagonal Line, equal to the Contents before found.

A R T. VII.

To find the Content of a Cask in the Form of the Frustrum of a Cone.

R U L E.

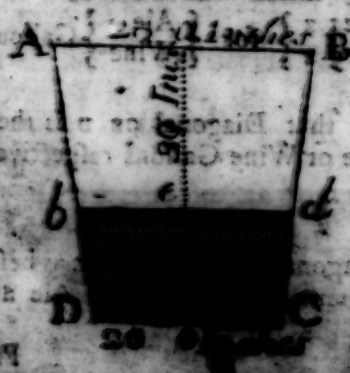
WITH some convenient Instrument take the Length of the Cask within the Heads, and also the Top and Bottom Diameters; then by Art. iv. Chap. viii. find the Content.

Or add the Top and Bottom Diameters together and take half that Sum for a Mean Diameter equal to that of a Cylinder of the same Height as the Cask; then by Art. ii. Chap. viii. find the Content, which will be near enough in Practice.

E X A M P L E.

Let the following Figure represent the Cask whose Top Diameter $A B = 28$ Inches, Bottom Diameter $D C = 20$ Inches, and the Length $E F = 30$ Inches: To find the Content in Ale Gallons.

A CONICAL CASE.

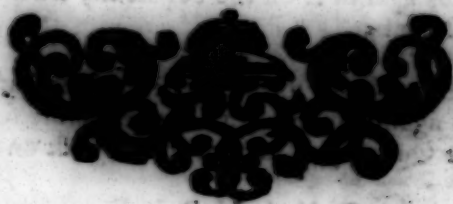


OPERATION.

By the Rule.

$AB = 28$ on D. on C. on D. on C.
 $DC = 20$ As 18.95 : 30 :: 24 : 48.2 = Ale Ga.
 — Ale gage-Lth. Mean. Content.
 $Sum = 48$ point.
 $\frac{1}{2} Sum = 24 = Mean.$

The Content thus found is = 48.1 Ale Gal-
 lons, which, with the Mean Diameter and Length,
 I have put into the Specimen of a Dimension Book
 exhibited in Chap. xiii.



CHAP. XII.

Of Ullaging.

DEFINITION.

ULLAGING of Casks is the finding of the Vacuity of a Cask that is Part full and Part empty: Or which is the same Thing, the finding of the Quantity of Liquor that is in such Cask.

ART. I.

To ullage a lying Cask by the Line of Segments on the Rule, having the Bung Diameter, wet Inches, and the Content of the Cask given.

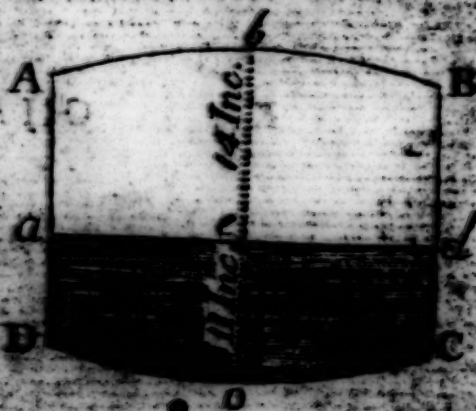
RULE.

1. **S**ET the Bung Diameter on *c* to 100 on the Line marked *S. ly.* or Segments lying; then seek the wet Inches on *c*, and observe what Number stands against it on the Segments, which call a fourth Number.
2. Set 100 on *A* to the Content of the Cask upon *B*, then against the fourth Number found on *A* is the Quantity of Liquor in the Cask on *B*.

EXAMPLE.

Let the following Figure represent the lying Cask, whose Bung Diameter $= b o = 25$ Inches, and wet Inches $= eo = 11$ Inches, and Content $= 48.3$ Gallons: To find the Quantity of Liquor in it, or the Part *aed* *con*.

A Lying Cask.



PROPORTION.

100 : 48.3 :: 11 : x

x = 5.33

5.33 : 48.3 :: 11 : 42.97

42.97 : 48.3 :: 11 : 45.3

ON A. ON B. ON A. ON B.

2. As 100 : 48.3 :: 41.4 : 20 = Quant. of Segment. Cont. Segmt. Cask. liq. in csk.

But if you wanted to know the Vacuity of the Cask, or what it wanted of being full, viz. the Part *abdea*, then, instead of making use of the wet Inches, use the dry Inches.

Thus,

on c. on S. l. on c. on S. l.

1. As 25 : 100 :: 14 : 58.2 = 4th Num.
B. Diam. Segs. Dry Inch. Segs.

ON A. ON B. ON A. ON B.

2. As 100 : 48.3 :: 58.2 : 28.1 = Vac. of Segment. Cont. Segmt. Vac. Cask.

This last Sum found added to the Quantity of Liquor in the Cask, viz. 20 + 28.1 = 48.1, which is but two Tenths less than the Content of the Cask.

ART. II.

To find the Ullage of a standing Cask by the Line of Segments on the Rule, having the Length, wet Inches, and the Contents of the Cask given.

RULE.

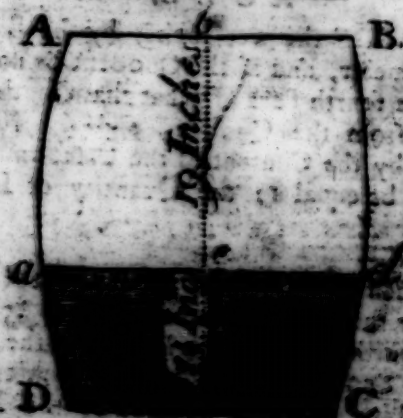
1. SET the Length of the Cask on c to 100 on the Line marked fl. or ft. or Segments standing; then seek the wet Inches on c and observe what Number stands against it on the Segments, which call a fourth Number.

Set 100 on A to the Content of the Cask upon B, then against the fourth Number before found on A is the Quantity of Liquor in the Cask upon B.

EXAMPLE.

Let the following Figure represent the standing Cask, whose Length *be* = 30 Inches, and wet Inches *re* = 11 Inches, and Content = 48.3 Gallons: To find the Quantity of Liquor in it, or the Part *abcd*.

A STANDING CASK.



PROPORTION.

on c. on fl. on c. on fl.
1. As 30 : 100 :: 11 : 35.2 = 4th Numbr.
Length. Segmt. W. Inchs. Segmts.

on A. on B. on A. on B.
2. As 100 : 48.3 :: 35.2 : 17 = Quantity of
Segmt. Cont. Segmt. Cont. liq. in cask.

But to find the Vacuity of the Cask or the Part
A B B D E A, the Proportion will stand thus:

on c. on fl. on c. on fl.
1. As 30 : 100 :: 19 : 64.3 = 4th Numbr.
Length. Seg. D. Inch. Seg.

on A. on B. on A. on B.
2. As 100 : 48.3 :: 64.3 : 31 = Vacuity
Segmt. Cont. Segmt. Cont. of Cask.

The Quantity of Liquor in the Cask = 17 +
31 = the Vacuity = 48 Gallons, nearly equal to
the Content of the Cask.

The Difference between the Sum of the sepa-
rate Parts thus found, and the whole Content of
the Cask, is occasioned by the Line of Segments
being adapted to one particular Sort of Casks only;
which is not very material, and may do well
enough in Practice.

A R T. III.

To ullage a lying Cask by the Pen, knowing the
Bung Diameter, wet Inches, and the Con-
tent of the Cask given.

R U L E.

1. **D**IVIDE the wet Inches by the Bung Dia-
meter, and if the Quotient be under .500
subtract a fourth Part of what that Quotient
of .500 from the Quotient, and the Remainder
multiply by the Content of the Cask, and the Pro-
duct will be equal to the Quantity of Liquor in
the Cask.

2. When the Quotient of the wet Inches di-
vided by the Bung Diameter exceeds .500, then
add a fourth Part of that Excess to the Quotient,
and that Sum multiplied by the Content of the
Cask will produce the Content of the Liquor in
the Cask.

EXAMPLE.

EXAMPLE.

Let the Diameter and Content of the Cask be the same as in Art. i. of this Chapter, viz. Bung Diameter = 25 Inches, wet Inches = 11, and the Content of the Cask = 48.3 Gallons: To find the Quantity of Liquor in the Cask.

OPERATION.

B.D. W. Inc. Quotient.

$$25 \text{) } 11.0 \text{ (} .44 = \text{less than } .500.$$

$$.500$$

$$.060 = \text{Difference.}$$

$$.015 = \frac{1}{4} \text{ of Diff. subtract.}$$

$$.425 = \text{Multiplier.}$$

$$48.3 = \text{Content of the Cask.}$$

$$1275$$

$$3400$$

$$1700$$

$$20,5275 = \text{Cont. of the Liq. in Cask.}$$

The Vacuity of the Cask may be found by dividing of the dry Inches by the Bung Diameter, instead of the wet, and managing the Quotient according to the foregoing Rules.

The Content of the Ullage found by the Pen differs somewhat from that found by the Line of Segments on the Rule. Which is nearest the Truth I will not take upon me to determine; however, the Line of Segments must be made use of by the Officers of the Excise for computing the Ullages of such Casks.

A R T. IV.

To find the Content of the Ullage of a standing Cask by the Pen, having the Length, wet Inches, and the Content of the Cask given.

R U L E.

1. **D**IVIDE the wet Inches by the Length of the Cask, and if the Quotient be under .500 subtract one tenth Part of what the Quotient wants of .500 from the Quotient.

2. If the Quotient exceeds .500 add one tenth Part of that Excess above .500 to the Quotient, then if the Difference in the first Case, or the Sum in the second be multiplied by the Content of the Cask, the Product will be equal to the Quantity of Liquor in the Cask.

EXAMPLE.

Let the Dimensions and Content of the Cask be the same as in Art. ii. of this Chapter, viz. Length = 30 Inches, wet Inches = 11, and the Content of the Cask = 48.3: To find the Quantity of Liquor in the Cask.

OPERATION.

Lth. W. Inc. Quotient.

$$30 \) \ 11.0 \ (\ .367 \text{ fere} = \text{less than } .500.$$

.500

.133 = Difference.

.0133 = $\frac{1}{10}$ of Diff. subtract.

.3537 = Multiplier.

48.3 = Content of the Cask.

10611

28296

14148

17,08371 = Cont. of Liq. in Cask

This agrees very well with the Ullage found by the Line of Segments in Art. ii. of this Chapter. If you make use of the dry Inches instead of the wet, and manage the Quotient as above, you will find the Vacuity of the Cask = 31.21629 Gallons.

ART. V.

To ullage a Cask in the Form of the Frustrum of a Cone. See the Figure in Art. vii. Chap. xi.

RULE.

TAKE the Top and Bottom Diameters, wet Inches, and the Length of the Cask; then by Art. x. Chap. viii. find the Diameter of the Cask at the Liquor's Surface; then having the Top and Bottom Diameters, and the Height of the little Frustrum *bedcon*, the Content may be found by Art. iv. Chap. viii. which will be equal to the Quantity of Liquor in the Cask. But in Practice you need only take the wet Inches, and guess at a Mean Diameter by laying the Gaging Rod over the Top of the Cask; and then find the Content by the Rule in the same Manner as if it were a Cylinder.

EXAMPLE.

Suppose the Depth of the Liquor = *eo* = 12.5 Inches, and I found the Mean Diameter = 22 Inches, what is the Content in Ale Gallons?

PROPORTION.

on D. on C. on D. on C.
As 18.75 : 12.5 :: 22 : 16.9 = Ale Gallons
A. Gage-pt. W. Inc. M. D. Cont.

CHAP.

C H A P. XIII. Of Gaging of Malt, &c.

A R T. I.

To gage and fix a Maltster's Square, or oblong Cistern.

R U L E.

1. **W**ITH a Dimension Cane, or some other convenient Instrument, measure the Length and Breadth of the Cistern in several Places, and in case you find any Variation add the Lengths or Breadths together and divide each Sum by the Number of Dimensions taken of each, and the Quotient will be a mean Length or Breadth; and at the same Time take also the Depth of the Cistern.

2. To find the Area of the Cistern multiply the Length by the Breadth, and that Product multiply or divide by the Factors, &c. for Square Malt Bushels in Page 49; and the Product or Quotient will be equal to the Area, which, with the Dimensions, transfer to the Specimen of a Dimension Book exhibited in Art. ix. of this Chapter.

E X A M P L E.

Suppose the Length of the Cistern was found to be = 114 Inches, the Breadth = 58.5 Inches, and the Depth = 36 Inches: To find the Area at one Inch, and fix it in the Dimension Book.

O P E R A T I O N.

Length = 114

Breadth = 58.5

570

912

570

2150.42)6669.00(3.10 = Area in Bushels.

645126

. 217740

215042

Rem. . . 26980

It will be necessary to prove your Work by the Rule in order to prevent any Mistake that may happen in the Operation by the Pen; and for that Purpose the Proportion will stand thus.

O N A. O N B. O N A. O N B.

As 2150.42 : 114 :: 58.5 : 3.10 as before.

M. B. Length. Breadth. Area.

The

The Answer comes out the same both by Pen and Rule, which proves the Work to be right: Therefore I transfer the same, with the Dimensions, to the Specimen of a Dimension Book in Art. ix. of this Chapter, which was required.

Note, If any Depth taken in this Cistern be multiplied by the Area, the Product will be equal to the Content of the Malt that is in it.

A R T. II.

To gage and fix a Maltster's Round Cistern, whether it be the Frustrum of a Cone, or Part of a Hogshead or Pipe standing upon one Head, the other being cut off.

R U L E.

1. **W**ITH some convenient Instrument take Mean Diameters between every six, or ten Inches of the Depth, according to the greater or lesser Difference of the Diameters of the Vessel, as was shewn in fixing of Victuallers Usensils. At the same Time take the Depth.

2. To find the Area of each Mean Diameter, square the Diameters and multiply or divide each Square by the Circular Factors. &c. for Malt Bushels in Page 49, and the Products or Quotients will be equal to the several Areas required.

E X A M P L E.

Suppose the Depth of the Cistern or Uting Fat to be gaged, &c. = 30 Inches, and the Diameter at 5 Inches from the Base = 29.6 Inches, at 15 Inches from the Base the Diameter = 33 Inches, and at 25 Inches from the Base the Diameter = 36.2 Inches: To find the respective Areas of the Mean Diameters in Malt Bushels.

O P E R A T I O N.

1 Mean Diameter = 29.6

29.6

—

1776

2664

592

—

2737.92)876,160(.32 = Lower Area.

821376

—

.547840

547584

—

Rem. . . . 256

In like Manner is the Area of 33 Inches found to be = .40, and the Area of 36 Inches = .48, as I have set them in the Specimen of a Dimension Book exhibited in the Ninth Article of this Chapter.

The Proportion by the Rule for the Areas is,

	on D.	on C.	on D.	on C.	
As	52.32	: 1 ::	29.6	: .32 =	1st Area.
	52.32	: 1 ::	33	: .40 =	2d Area.
	52.32	: 1 ::	36.2	: .48 =	3d Area.
C. Gage-pt.	Unity.	M. D.			Area.

Note, all Depths that are taken in this Cistern must be multiplied by the respective Areas to which they belong, viz. If 10 Inches or under by the 1st Area, that Part from 10 to 20 by the 2d Area, and from 20 to 30 Inches of the Depth by the 3d Area, which added together will be the Content at any Depth.

A R T. III.

To gage a Couch of Malt in a Square or Oblong Frame, and find the Content of the same.

R U L E.

1. **W**ITH a Box and Tape measure the Length and Breadth of the Frame, and with a Gaging Rod take the Depth of the Malt in several Places; then add these Depths together and divide their Sum by the Number of Depths taken, and the Quotient will be a Mean Depth.

2. To find the Content multiply the Length, Breadth, and Depth together, and that Product divide by the Square Divisor for Malt Bushels in Page 49, and the Quotient will be equal to the Content of the Malt in the Couch.

E X A M P L E.

Suppose the Length of a Couch of Malt was found = 105 Inches, the Breadth = 104 Inches, and the Depth = 20 Inches: To find the Content in Bushels.

O P E R A T I O N.

Length = 105

Breadth = 104

—

220

1050

—

10920

20

—

Product = 218400

PRODUCT.

P R O D U C T.

2150.42)218400,00(101.5=Cont. in Bush.
215042

.335800

215042

1207580

1075210

Rem. 132370

To find the Content by the Rule the Proportion is

ON M.	ON B.	ON A.	ON B.	
As 20	: 105	:: 104	: 101.5	= as before found.
Depth.	Length.	Breadth.	Cont.	

The common Method of gaging Malt in Frames is to find the Area of the Frame at one Inch deep, and to keep that Area fixed in the Malt Book. Then the Content of the Couch is easily found; for if you multiply that Area by any Depth the Product will be the Content.

A R T. IV.

To gage a Couch of Malt that is not in a Frame, but laid in a Square or Oblong Form, with one or more of its Sides slanting.

R U L E.

1. IF the Couch is laid in the Middle of the Floor with both the longest and shortest Sides slanting: Then fasten the Tape at the Edge of the Top Surface and draw out the Tape till it comes perpendicular over the outside Edge of the Bottom on the other Side, which may be found by dropping of a few Corns of Malt and catching the Tape to the Place where the Corns fall from, so as to touch the outside Edge of the Bottom.

This do for both Length and Breadth, which Distances will be equal to the true Length and Breadth; for the slanting Part that is cut off on one Side will just fill up the empty Space on the other Side.

2. If one of the Sides, or one of the Ends are laid against a Wall that is perpendicular to the Floor, then fix the End of the Tape close to the Wall on the Surface of the Malt and draw out the other End till it comes perpendicular over the outside Edge of the Bottom; then lay hold of the Tape with your finger and Thumb, and keep it back again till it just touches the outside Edge of the

the Top Surface, with the other Hand draw the Tape tight over the Malt, and observe the Number of Inches at the Place of its doubling, which will be equal to the Length or Breadth of the Couch.

3. Take the Depth and find the Content according to the Directions in the last Article, which is so easy that I shall pass over this Article without giving of any Example.

A R T. V.

To gage a Couch of Malt in Form of the Frustrum of a Cone.

IN some Countries there are private Maltsters that steep so small a Quantity as four or five Bushels of Barley; and it is the common Way of such Maltsters to empty their Cisterns in the Middle of the Floor by throwing it all on a Heap, whereby there is formed a little Cone, and in that Posture they leave it for the Gager to take his Dimensions.

R U L E.

1. With the Gaging Rod strike off the Top of the Cone and make it level on the Top; then take the Depth of the Couch in the Middle and fix the End of the Tape on the outside Edge of the Top Diameter, and draw out the other End across over the Malt till it reaches perpendicularly over the outside Edge of the Bottom Diameter, then that Distance will be equal to half the Sum of the Top and Bottom Diameters, which may be taken for a Mean Diameter, tho' not exactly true; yet it may serve well enough in Practice for such small Quantities of Malt.

2. To find the Content of the Couch square the Mean Diameter thus found and multiply that Square by the Depth of the Malt, that Product divide by the Circular Divisor for Malt Bushels in Page 49, and the Quotient will be the Content.

But this may be found near enough the Truth by the Rule, and with much more Ease and Expedition; therefore I shall omit the working of this Example by the Pen.

E X A M P L E.

Suppose the Depth of the Couch was found

to be = 6 Inches, and the Mean Diameter = 50 Inches, what will its Content be in Bushels?

Proportion by the Rule.

ON D. ON C. ON D. ON C.

As 52.32 : 6 :: 50 : 5.5 = Cont. in Bush.
 Gage-pt. Depth. M. D. Content.
 M. R.

A R T. VI.

To gage a Floor of Malt, and to find the Content of the same.

1. **F**LOORS of Malt are generally laid in a Square or Oblong Form with their Surfaces uneven, that is, in some Places deeper than others; therefore with a Gaging Rod and brass Plate take the Depth in divers Places, which several Depths add together and their Sum divide by the Number of Depths taken, and the Quotient will be equal to a Mean Depth. Then with a Box and Tape measure the Length and Breadth of the Malt and enter the Dimensions in your Book.

2. To find the Content of the Floor of Malt: This may be found in the same Manner as the Couch in Art. iii. of this Chapter, both by Pen and Rule, and if the Malt Line marked M. D. is very good, by the Rule itself, if the Amount of the Floor be under 300 Bushels. But if it is not true, and the Floor be large, it will be best to use both Pen and Rule for finding its Content, according to the following Directions.

C A S E I.

If the Length or Breadth of the Floor be exactly 215 Inches, then multiply the other Dimension by the Depth, and the Product will be the Content in Bushels. But you must mind to prick off one Figure for a Decimal when there is none in the Depth; and if there is a Decimal taken in the Depth, then prick off two Figures in the Product for Decimals.

C A S E II.

If the Breadth of the Floor be above 215 Inches, then subtract 215 from it, and the Remainder

Of Malt Gaging.

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der reserve. Next multiply the Length by the Depth of the Floor, and prick off one or two Decimals from the Product, as in the first Case.

To this Product add the Content of a Floor whose Dimensions are equal to the Length and Depth of the given Floor, and Breadth equal to the reserved Number, or equal to the Excess of the Floor's Breadth above 215 (which must be found by the Rule) and the Sum will be equal to the Content of the Floor in Bushels, &c.

C A S E III.

If the Breadth of the Floor be less than 215 Inches subtract it from 215, and reserve the Remainder; then, as in the last Case, multiply the Length by the Depth, and prick off the Decimals by the first Case; then from the Product subtract the Content of a Floor whose Dimensions are equal to the Length and Depth of the given Floor, and the Breadth equal to the reserved Number, or equal to what the Breadth wants of 215 (which is found by the Rule) and the Remainder will be equal to the Content of the Floor in Bushels, &c.

E X A M P L E I.

Suppose with a Gaging Rod and
brass Plate I took ten Depths, and they
were as they stand in the Margin, whose
Sum = 35 Inches, which divided by
10 = 3.5 = the Mean Depth; and
suppose the Length of the Floor =
650 Inches, and its Breadth = 250
Inches, what will the Content be in
Malt Bushels?

1 = 3.2
2 = 3.6
3 = 2.5
4 = 3.0
5 = 4.0
6 = 4.5
7 = 3.7
8 = 3.5
9 = 4.1
10 = 2.9
Sum = 35.0
10 = 3.5

O P E R A T I O N.

Length.	Breadth.	Depth.
650	250	3.5
3.5	215	
3250	35 = Rem ^r . above 215.	
1950		
Prod. = 227,50		
Add = 37.1		
Cont. = 264.6 Bushels.		

Then by the Rule.

ON M.D.	ON B.	ON A.	ON B.
As 3.5	: 650	:: 35	: 37.1 = Content to be
Depth.	Length.	Diff.	Content. added.

As the Rule now stands you may prove your Work; for if, instead of looking against 35 on A, you look against 250 = the Breadth on A, you will find 264.6 on B = Content before found, which proves the Work to be done right.

EXAMPLE II.

Let the Length of a Floor = 700 Inches, Breadth = 200 Inches, and the Depth = 4 Inches: To find the Content in Bushels.

OPERATION.

Length.	Breadth.	Depth.
700	200	4
4	215	
Product = 280.0	15 = Diff. less than 215.	
Sub. = 19.5		
Content = 260.5		

Then by the Rule.

ON M.D.	ON B.	ON A.	ON B.
As 4	: 700	:: 15	: 19.5 = Cont. to be
Depth.	Length.	Diff.	Cont. subtracted.

As the Rule now stands, against 200 on A is 260.5 on B equal to the Content in Bushels, as above found, which proves the Work to be done right. I have given no Example in the first Case, because it is so easy that it would be of no Use.

ART. VII.

To find the Content of a Floor of Malt (having the Dimensions given) without either Pen or Rule.

1. IF either the Length or Breadth = 215 or 21.5 the other is the Area, allowing one Figure in the first, and two in the second Case for Decimals.

But if neither the Length or the Breadth be equal to one of the above Numbers, but agree with any of the following Numbers, viz.

If the Length or Breadth =	$\left\{ \begin{array}{l} 430 \text{ or } 43 \\ 645 \text{ or } 64.5 \\ 860 \text{ or } 86 \\ 107.5 \\ 129 \\ 150 \\ 172 \\ 1935 \end{array} \right\}$	then multiply the other Dimension by	$\left\{ \begin{array}{l} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array} \right\}$	and the Product will be the Area.
				Which

Which Area multiplied into the Depth will produce the Content of the Floor, allowing for Decimals in the Product, according to the foregoing Directions.

These Numbers are Multiples of 215, viz. $430 = 215 \times 2$, and $645 = 215 \times 3$, &c. therefore may easily be remembered; and thereby the Content of any Floor of Malt may be found almost by Inspection.

2. If it should happen that neither the Length nor the Breadth agree with any of the foregoing Numbers, but are nearly equal to one another, then you may take from one of them as many as will make the other equal to one of the foregoing Numbers, and multiply the Remainder by the Multiplier belonging to the Number it was compared to, and the Product will be equal to the Area, as before.

3. If the Length be double the Breadth, what you take from the Length must be but half added to the Breadth; or if you take from the Breadth and add to the Length, add double the Number to the Length of what you took from the Breadth, and so in Proportion for any other.

A R T. VIII.

To find the Area of a Floor of Malt, by having the Roots of the Length and Breadth; and also the Content, having its Depth in Inches given.

IN order to find the Roots of the Length and Breadth you must have the Root of every Bushel, and every Tenth of a Bushel, marked on the Tape; which may be done by the Help of the following Table of Roots; for if at the Inches and Tenths, as they stand in the Column of Inches, you mark the Bushels and Tenths, as they stand against them in the Column of Roots on the Tape, you may thereby find the Root of any Length or Breadth from 23,2 Inches to 700 2 Inches.

" The Table is constructed in the following Manner: The Square Root of 2150.42, the solid Inches in a Bushel of Malt is $= 46.37$, which is also $=$ to the Root of 1 Bushel. Now if this Number be doubled and trebled, &c. you will also double and

L

treble

treble the Roots; for if at 46.37 is the Root of 1 Bushel, then at $46.37 \times 2 = 92.7$ is the Root of 2 Bushels, and at $46.37 \times 3 = 139.1$ is the Root of 3 Bushels; and thus, by continually adding of 46.37 for every Bushel, you may gain as many Roots as you please of the even Bushels, and the Roots of the Tenths of a Bushel are gained by adding and subtracting one Tenth of 46.37 to, and from that Sum for every Tenth of a Bushel.

E X A M P L E.

Root of 1 Bush. = 46.37	Root of 1 Bush. = 46.37
$\frac{1}{10}$ of 46.37 add = 4.637	$\frac{1}{10}$ of 46.37 sub. = 4.637
Root of 1.1 B. = 51.007	Root of .9 B. = 41.733
4.637	4.637
Root of 1.2 B. = 55.644	Root of .8 = 37.096
4.637	4.637
Root of 1.3 B. = 60.281	Root of .7 = 32.459

And thus, by continuing the Work, was the following Table of Roots completed.

Now by having these Roots marked on the Backside of the Tape, as they stand in the Table, it will be an easy Matter to find the Area of any Floor of Malt.

For if the Roots of the Length and Breadth of the Floor are taken with a Tape thus marked, and multiplied together, the Product will be the Area of that Floor; which Area multiplied by the Depth will produce the Content; which may be performed by the Head without either Pen or Rule.

E X A M P L E.

Suppose I found the Root of the Length of a Floor = 10.2 Bushels, and the Root of the Breadth = 3.3 Bushels, and the Depth = 4 Inches: To find the Content.

$10.2 \times 3.3 = 33.66 = \text{Area, which multiplied by 4} = 134.6 = \text{Contents in Bushels.}$

A Table of Roots.

Inch.	Rts.	Inch.	Rts.	Inch.	Rts.	Inch.	Rts.	Inch.	Rts.
23.2	.5	162.3	.5	301.4	.5	440.5	.5	579.6	.5
27.8	.6	166.9	.6	306.	.6	445.2	.6	584.3	.6
32.5	.7	171.6	.7	312.7	.7	450.	.7	588.9	.7
37.1	.8	176.2	.8	315.3	.8	454.4	.8	593.	.8
41.7	.9	180.8	.9	320.	.9	459.1	.9	598.2	.9
46.4	1.B.	185.5	1.B.	324.6	1.B.	463.7	10.B.	602.8	13.B.
51.	.1	190.1	.1	329.2	.1	468.3	.1	607.4	.1
55.6	.2	194.8	.2	333.9	.2	473.	.2	612.1	.2
60.3	.3	199.4	.3	338.5	.3	477.6	.3	616.7	.3
64.9	.4	204.	.4	343.1	.4	482.2	.4	621.4	.4
69.6	.5	208.7	.5	347.8	.5	486.9	.5	626.	.5
74.2	.6	213.3	.6	352.4	.6	491.5	.6	630.6	.6
78.8	.7	217.9	.7	357.	.7	496.2	.7	635.3	.7
83.5	.8	222.6	.8	361.7	.8	500.8	.8	640.	.8
88.1	.9	227.2	.9	366.3	.9	505.4	.9	644.5	.9
92.7	2.B.	231.8	5.B.	371.	3.B.	510.1	11.B.	649.	14.B.
97.4	.1	236.5	.1	375.6	.1	514.7	.1	653.8	.1
102.	.2	241.1	.2	380.2	.2	519.3	.2	658.5	.2
106.7	.3	245.8	.3	384.9	.3	524.	.3	663.1	.3
111.3	.4	250.4	.4	389.5	.4	528.6	.4	667.7	.4
115.9	.5	255.	.5	394.1	.5	533.3	.5	672.4	.5
120.6	.6	259.7	.6	398.8	.6	537.9	.6	677.	.6
125.2	.7	264.3	.7	403.4	.7	542.5	.7	681.6	.7
129.8	.8	268.9	.8	408.1	.8	547.2	.8	686.3	.8
134.5	.9	273.5	.9	412.7	.9	551.8	.9	690.9	.9
139.1	3.B.	278.2	6.B.	417.3	5.B.	556.4	12.B.	695.5	15.B.
143.7	.1	282.9	.1	422.	.1	561.1	.1	700.2	.1
148.4	.2	287.5	.2	426.6	.2	565.7	.2		
153.	.3	292.1	.3	431.2	.3	570.4	.3		
157.7	.4	296.8	.4	435.9	.4	575.	.4		

A R T. IX.

A Table of the Dimensions, and Content in cubic Inches, of the Standard Bushel, Half Bushel, Peck, and Gallon, with the Avoirdupoise Weight of Water that each will contain.

A Table.

Names.	Diam.	Depth.	Cubic Inches.	Avoirdupoise.		
				lb.	oz.	Drs.
Bushel.	18.5	8.0	2150.42	77	12	8.60
Half Bushel.	14.7	6.34	1075.21	38	14	4.30
Peck.	11.7	5.0	537.60	19	7	2.15
Gallon.	9.3	3.96	268.80	9	11	9.07

By this Table it will be easy to make any of the Measures therein mentioned, or to try such as are already made by a good Pair of Scales.

N. B. It is not material whether or no the Dimensions of the Bushel be the same with the Standard, provided the Capacity of it be the same both in Weight and Measure.

A Specimen of a Dimension Book.

40.0 1 MT	30.0 2 MT	38.0 Cop.	Area	10.0 ⊙ U B	Back	40.0 1 Rd	40.0 2 Rd	Area
60.2	60.0	50.0	6.96	T 72.0	L 91.3	50.0	67.5	12.70
	60.0	48.5	6.55	C 50.0	B 59.7		62.6	10.88
		46.5	6.02				57.7	9.27
12.54	11.86	44.3	5.46	10.03	19.33	6.96	52.8	7.76

24.0 3 Rd	Area	30.0 4 R	⊙ Area	30.0 1 Sq.	Area
24.9	1.73	T 44.4	4.75	Sq. { 42.5 }	6.41
26.5	1.96	C 38.4		Sq. { 42.5 }	
25.0	1.74	T 43.4	4.41	Sq. { 38.5 }	5.25
24.0	1.60	C 36.5		Sq. { 38.5 }	
		T 42.2	4.02	Sq. { 32.5 }	3.74
		C 34.2		Sq. { 32.5 }	

The Dimensions, &c. of Casks.

Nº.	H.	B.	L.	Con.	Nº.	H.	B.	C.	Con.
1	23.0	26.5	28.3	51	6	2 V.			
	2 V.					24.5	31.5	42.0	98
2	23.0	26.5	28.3	50		3 V.			
	3 V.				7	24.5	31.5	42.0	95
3	23.0	26.5	28.3	49		4 V.			
	4 V.				8	24.5	31.5	42.0	92
4	23.0	26.5	28.3	48	9	Mn. 24.0	30.0		48
5	24.5	31.5	42.0	101					

Maltsters Cisterns.

Depth	Length	Breth.	Area
36.0	114.	58.5	3.10
{	10 Diam.	36.2	.48
	10 Diam.	33.0	.40
	10 Diam.	29.6	.32
30			

The above Specimen was all collected from the Operations in Chapters ix. xi. and xiii.

C H A P. XIV.

Tables of Rates, &c.

THIS Chapter contains the Tables of Rates and Measures of all exciseable Commodities, very useful for the moneying of the Goods, and making out of the Charges in the Vouchers and Abstracts.

1. *A Table of Rates and Measures of exciseable Liquors in the Country.*

Species, and by what charged.	N ^o . of Galls. each con- tains of		Rates.		
	Beer	Wine	L.	s.	d.
Strong Beer per Barrel	34	—	0	5	0
Small Beer per ditto	34	—	0	1	4
Sweets per ditto	—	3 ¹ / ₂	0	12	0
Vinegar per ditto	34	—	0	8	9
Mum per ditto	—	32	1	5	0
Cyder per Hogsh. Exc. 6s. 8d. }	—	63	0	10	8
Ditto for Malt Duty 4s. 0d. }	—	63	0	6	8
Verjuice per Hoghead	—	1	0	0	11
Mead or Metheglin per Gallon	—	1	0	0	6
Low Wines from Melasses per Gallon	—	1	0	1	0
Spirits per Gallon from ditto	—	1	0	1	0
Low Wines from foreign Wines or Cyder imported s. d. per Gallon	—	1	0	1	3
New Duty 0:2	—	1	0	1	3
New additional Duty 0:6					
Further add. Duty 0:3					
Spirits from for. Wine, or Cyder imported, per Gallon	—	1	0	1	3
New Duty 0:6	—	1	0	1	3
New additional Duty 0:3					
Low Wines from malted Corn, Wash and Tilts per Gallon	—	1	0	0	4
New Duty 0:1	—	1	0	0	4
New additional Duty 0:0					
Further add. Duty 0:1					
Spirits from malted Corn, Wash and Tilts, per Gallon	—	1	0	1	0
New Duty 0:3	—	1	0	1	0
New additional Duty 0:1					
Further add. Duty 0:4					
Low Wines from Cyder or any other English Materials per Gallon	—	1	0	0	5 ¹ / ₂
New Duty 0:3	—	1	0	0	11
New additional Duty 0:0					
Further add. Duty 0:1					
Spirits from Cyder, or any other English Materials per Gall.	—	1	0	0	11
New Duty 0:3	—	1	0	0	11
New additional Duty 0:1					
Further add. Duty 0:3					

2. A Table of the Rates of Candles, Soap, &c.

Species, and by what charged.	Rates.		
	L.	s.	d.
Candles, Tallow	0	0	1
Ditto, Wax	0	0	8
Soap ———	0	0	1 1/2
Starch ———	0	0	2
Hops ———	0	0	1

} per Pound.

3. A Table of the Rates of Printed Linens, &c.

Species, and by what charged.	Rates.		
	L.	s.	d.
Printed Silks half Yard wide per Yard	0	1	0
Silk Handkerchiefs Yard square	0	0	4
Calicoes per Yard ———	0	0	6
Linen Stuffs per Yard ———	0	0	3

4. A Table of the Rates of Coffee, &c.

Species, and by what charged.	Rates.		
	L.	s.	d.
Coffee	0	2	0
Chocolate	0	1	6
White Glass	0	9	4
Green ditto	0	2	4

} per Pound.

} per C. Wt. of Metal.

5. A Table of the Rates of Wrought Plate, &c.

Species, and by what charged.	Rates.		
	L.	s.	d.
Wrought Plate	0	0	6
Silver Wire	0	0	6
Gilt Wire	0	0	2

} per Ounce Troy.

6. A Table of the Rates of Hides, &c.

How dressed.	Species, and how charged.	Rates.		
		L.	s.	d.
Tanned.	Sheep and Lamb			
	Butts and Backs			
	Hides ——— } per Pound.		0	1
	Calves and Kipps			
	Hogs and Dogs			
	Roans per Pound ———	0	0	2
Tawed.	Goats tanned with Shomack per ditto	0	0	4
	Skins and Pieces ad Valorem per Cent.	30	0	0
	Sheep and Lamb per Pound ———	0	0	1
	Calves and Kipps per ditto ———	0	0	1
	Buck and Doe per ditto ———	0	0	6
	Calves and Slink without Hair } per			
	Dog and Kid ——— } Doz.	0	3	0
	Beaver and Goat per ditto ———	0	2	0
Draft'd in Oil.	Calves Slink with Hair per ditto	0	3	0
	Horse Hides per Hide ———	0	3	6
	All other Hides per ditto ———	0	3	0
	Skins and Pieces, &c. ad Valorem			
	per Cent. ———	30	0	0
	Sheep and Lamb per Pound ———	0	0	3
Vellum per Dozen ———	Hides, Deer, } per ditto.	0	0	6
	Goat and Beaver } per ditto.	0	0	8
	Calve Skins per ditto ———	0	0	2
	Skins and Pieces, &c. ad } per ditto.	0	0	2
	Valorem ——— } per Cent.	30	0	0
Parchment per ditto ———		0	3	0
		0	3	6

7. Tables of Equal Parts at 15l. and 30l. per Cent.

At 15 l. per Cent.				At 30 l. per Cent.			
Value	Duty	Value	Duty	Value	Duty	Value	Duty
s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.
0 50	0 7	10 05	1 6	20 0	3 0	30 0	4 6
0 100	1 4	10 10	1 10	20 5	3 5	30 5	5 1
1 30	2 1	11 3	1 17	21 10	3 15	31 10	5 6
1 80	3 1	12 8	1 24	22 0	3 30	32 0	6 0
2 10	4 1	13 1	1 31	22 5	3 35	32 5	6 5
2 60	5 1	14 6	1 38	23 10	3 40	33 10	7 0
2 110	6 1	15 11	1 45	24 0	3 45	34 0	7 5
3 40	7 1	16 4	1 52	25 5	3 50	35 5	8 0
3 90	8 1	17 9	2 0	26 10	3 55	36 10	8 5
4 20	9 1	18 14	2 7	27 0	4 0	37 0	9 0
4 70	10 1	19 19	2 14	28 5	4 5	38 5	9 5
5 00	11 1	20 0	2 21	29 10	4 10	39 10	10 0
5 50	12 1	20 5	2 28	30 0	4 15	40 0	10 5
5 100	13 1	21 10	2 35	31 5	4 20	41 5	11 0
6 30	14 1	22 15	2 42	32 10	4 25	42 10	11 5
6 80	15 1	23 11	2 49	33 0	4 30	43 0	12 0
7 10	16 1	24 16	2 56	34 5	4 35	44 5	12 5
7 60	17 1	25 11	3 0	35 10	4 40	45 10	13 0
7 110	18 1	26 16	3 7	36 0	4 45	46 0	13 5
8 40	19 1	27 11	3 14	37 5	4 50	47 5	14 0
8 90	20 1	28 16	3 21	38 10	4 55	48 10	14 5
9 20	21 1	29 11	3 28	39 0	5 0	49 0	15 0
9 70	22 1	30 16	3 35	40 5	5 5	50 5	15 5
10 00	23 1	31 0	3 42	41 10	5 10	51 10	16 0

8. A Table of the Rates of Paper, &c.

Species.	By what charged.	Rates.		
		L.	s.	d.
Demy Fine	per Ream	0	23	
Ditto Second	per ditto	0	16	
Crown Fine	per ditto	0	16	
Ditto Second	per ditto	0	11	
Fool's Cap Fine	per ditto	0	16	
Ditto Second	per ditto	0	11	
Fine Pot	per ditto	0	16	
Ditto Second	per ditto	0	09	
Brown Large Cap	per ditto	0	09	
Ditto Small Ordinary	per ditto	0	06	
Whited Brown	per Bundle	0	09	
Paste Boards, Mill Boards, and Scale Boards	per C. Wt.	0	46	
Printed, Painted, or Stained Paper for Hangings	per Yard square	0	01	

All other Papers, white or brown, of whatsoever Kind or Colour, are charged ad Valorem at 18 l. per Cent.

Note, A Bundle of Paper contains two Reams.

9. A Table of Equal Parts at 18 l. per Cent. ad Valorem.

Val.	Duty	Val.	Duty	Val.	Duty	Val.	Duty
s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	l. s. d.	l. s. d.
0 1 1/2	0 0 1	0 9	0 1 1/2	10 0 1	9	0 17 6	3 1 1/2
0 2	0 0 2	0 9 1/2	0 1 10	10 6	10	0 18 0	3 3
0 2 1/2	0 0 3	0 10	0 1 11	11 0	11	0 18 6	3 4
0 3	0 0 4	0 10 1/2	0 2	11 6	0	0 19 0	3 5
0 3 1/2	0 0 5	0 11	0 2 1/2	12 0	3	0 19 6	3 6 1/2
0 4	0 0 6	0 11 1/2	0 3	12 6	3	1 0 0	3 7 1/2
0 4 1/2	0 0 7	1 0	0 3 1/2	13 0	4	2 0 0	7 2 1/2
0 5	0 0 8	1 0 1/2	0 4	13 6	5	3 0 0	10 9 1/2
0 5 1/2	0 0 9	1 1	0 4 1/2	14 0	6	4 0 0	14 4 1/2
0 6	0 0 10	1 1 1/2	0 5	14 6	7	5 0 0	18 0
0 6 1/2	0 0 11	1 2	0 5 1/2	15 0	8	6 0 0	1 7 1/2
0 7	0 0 12	1 2 1/2	0 6	15 6	9	7 0 0	5 2 1/2
0 7 1/2	0 0 13	1 3	0 6 1/2	16 0	10	8 0 0	8 9 1/2
0 8	0 0 14	1 3 1/2	0 7	16 6	11	9 0 0	12 4 1/2
0 8 1/2	0 0 15	1 4	0 7 1/2	17 0	12	10 0 0	16 0

10. A Table of Wine Measure.

Names.	Cubic Inches.	Avoirdupois Weight.			Pints.	Quarts.	Gallons.	Rundlets.	Barrels.	Tierces.	Hogheads.	Punches.	Butts.	Tons.
		lb.	oz.	dr.										
Ton	58212	2042	9	4	2016	1008	252	14	8	6	4	3	2	1
But	29106	1021	4	10	1008	504	126	7	4	3	2	1	1	1
Punchon	29404	620	13	12	504	252	63	4	2	1	1	1	1	1
Hoghead	14553	510	10	5	252	126	31	2	1	1	1	1	1	1
Tierce	9722	340	6	14	126	63	15	1	1	1	1	1	1	1
Barrel	7276.5	252	6	10	63	31	7	1	1	1	1	1	1	1
Rundlet	4158.	141	14	6	31	15	3	1	1	1	1	1	1	1
Gallon	331.	8	1	1	7	3	1	1	1	1	1	1	1	1
Quart	57.75	2	0	6	1	1	1	1	1	1	1	1	1	1
Pint	28.875	1	0	3	1	1	1	1	1	1	1	1	1	1

11. A Table of London Beer Measure, 36 Gallons to a Barrel.

Names.	Cubic Inches.	Avoirdupois Wt. of London Beer.			Pints.	Quarts.	Gallons.	Firkins.	Kilderkins.	Barrels.
		lb.	oz.	dr.						
Barrel	10152	387	0	0	288	144	36	4	2	1
Kilderkin	50.76	193	8	0	144	72	18	2	1	1
Firkin	538	96	12	0	72	36	9	1	1	1
Gallon	282	50	12	0	36	18	4	1	1	1
Quart	70.5	12	11	0	18	9	1	1	1	1
Pint	35.25	6	5	8	9	4	1	1	1	1

12. A Table of London Ale Measure, 32 Gallons in a Barrel.

Names.	Cubic Inches.	Avoirdupois Wt. of Col- lege Ale.			Pints.	Quarts.	Gallons.	Firkins.	Kilderkins.	Barrels.
		lb.	oz.	dr.						
Barrel	9024.	331	6	15	256	128	32	4	2	1
Kilderkin	4512	165	11	7	128	64	16	2	1	1
Firkin	2256	82	13	11	64	32	8	1	1	1
Gallon	282	10	5	14	8	4	1	1	1	1
Quart	70.5	2	9	6	2	1	1	1	1	1
Pint	35.25	1	4	11	1	1	1	1	1	1

13. *A Table of Beer and Ale Measure for the Country, 34 Gallons in a Barrel.*

Names.	Cubic Inches.	Avoirdupoise Wt. of clear Water			Pints.	Quarts.	Gallons.	Firkins.	Kilderkins.	Barrels.
		lb.	oz.	dr.						
Barrel	9588	346	12	8½	272	136	34	4	2	1
Kilderkin	4794	173	6	4	136	68	17	2	1	
Firkin	2397	86	18	2	68	34	8½	1		
Gallon	282	10	3	3	8	4	1			
Quart	70.5	2	8	12½	2	1				
Pint	35.25	1	4	6½	1					

14. *A Table of Specific Gravity, one Ounce being the Integer.*

Names.	Ounces and		Ounces and	
	decim. Parts Troy.		decim. Parts Avoirdup.	
Fine Gold is	10.3	9273	11.36	5602
Standard ditto	9.96	2623	10.93	0422
Quicksilver	7.38	4431	8.10	1753
Lead	5.98	4010	6.55	3835
Fine Silver	5.89	0035	6.41	8324
Standard ditto	5.55	0769	6.09	6596
Rose Copper	4.74	7121	5.20	8369
Plate Brass	4.40	4273	4.83	2116
Cast Brass	4.27	2409	4.63	0300
Steel	4.14	3127	4.54	4503
Common Iron	4.03	1361	4.42	2979
Block Tin	3.86	1519	4.23	6638
Fine Marble	1.42	9411	1.56	8239
Common Glass	1.36	0843	1.49	3937
Mustard	0.98	8456	1.08	4477
Dry Ivory	0.96	2083	1.05	5342
London brewed Ale	0.95	6878	0.60	9929
Beer Vinegar	0.94	5392	0.59	1070
Rack	0.94	3864	0.59	0906
Dry Box Wood	0.94	3864	0.59	6037
Milk	0.94	3864	0.58	9362
Urine	0.94	3864	0.58	8791
College Plain Ale	0.94	3864	0.58	7648
Salt Water	0.94	3864	0.59	4894
Pump ditto	0.92	7458	0.57	8697
Claret	0.92	3760	0.57	4606
Lipfeed Oil	0.49	1991	0.53	9345
Proof Spirits of Brandy	0.48	9868	0.53	6736
Sound Dry Oak	0.48	9008	0.53	6569
Oil Olive	0.48	1369	0.52	8350
Oil of Turpentine	0.38	3880	0.41	5820
Half Pint of Mustard Seed	5.54	3518	6.08	2031

15. A Table of the Avoirdupois Weight of a Cubic Foot of several Bodies in Pounds and Decimal Parts.

Names.	Lb.	lb.	oz.	gr.
Gravel	109.3125	109	5	0
Common Sand	85.25	85	4	0
Newcastle Coal	67.75	67	12	0
Pump Water	62.734	62	12	6
Wood Ashes	58.312	58	5	0
Bay Salt	54.062	54	1	0
White Pease	50.5	50	8	0
Field Beans	50.5	50	8	0
Wheat of the best Sort	48.5	48	8	0
White Sea Salt	43.75	43	12	0
Barley	41.125	41	2	0
Wheaten Meal unfifted	31.	31	0	0
Malt 2 Months old	30.25	30	4	0
White Oats	29.5	29	8	0
Rye Meal unfifted	28.	28	0	0

The Proportion of the Standard Avoirdupois Weight to the Standard Troy Weight have been found by nice Experiments made for that Purpose to be as follows, viz. that 15 lb. of Avoirdupois is equal to 18 lb. 2 oz. and 15 pwt. Troy; so that 140 Ounces of Avoirdupois is equal to 128.75 Ounces of Troy; which in the least Terms in whole Numbers is as below:

$$144 \text{ lb. of Avoirdupois} = 175 \text{ lb. of Troy.}$$

$$192 \text{ oz. of ditto} = 175 \text{ oz. of ditto.}$$

By these Proportions any Numbers of Pounds or Ounces of Avoirdupois may be reduced to Troy, and *e contra* any Number of Pounds or Ounces of Troy may be reduced to Avoirdupois: As for Example,

In 28 lb. of Avoirdupois, how many Pounds of Troy?

The Proportion both by Pen and Rule is,

$$\begin{array}{ccccccc} \text{ON A.} & \text{ON B.} & \text{ON A.} & \text{ON B.} & & & \\ \text{As } 144 & : 175 & :: 28 & : 34.0278 = \text{Troy Wt.} \\ \text{Avoir. lb.} & \text{Troy lb.} & \text{Av. lb.} & \text{Troy lb.} & & & \end{array}$$

Another Example.

In 16 oz. of Avoirdupois how many Ounces of Troy?

PROPORTION.

$$\begin{array}{ccccccc} \text{ON A.} & \text{ON B.} & \text{ON A.} & \text{ON B.} & & & \\ \text{As } 192 & : 175 & :: 16 & : 14.58 = \text{Answer.} \\ \text{Avoir. oz.} & \text{Troy oz.} & \text{Av. oz.} & \text{Troy oz.} & & & \end{array}$$

And

And in this Manner may the Avoirdupois Pounds of any of the Bodies mentioned in the last Table be reduced to Troy Weight.

And on the contrary, by transposing the first and second Terms any Number of Pounds and Ounces may be reduced from Troy to Avoirdupois Weight.

The chief Use of the Tables of specific Gravity is to try Weights and Measures; for by knowing how many Inches are contained in a solid Body we can find the Weight of that Body; or by having the Weight of that Body we can find how many solid Inches it contains.

For Example, What is the Avoirdupois Weight of a Winchester Pint of Pump Water?

The Proportion is, As Unity is to the tabular Number, so are the solid Inches contained in that Body to its Weight.

ON B.	ON A.	ON B.	ON A.	lb. oz. dr.
As 1 :	.578697 ::	35.25 :	20.399 =	1 : 4 : 6
Unity.	Tab. Num.	Sol. Inc.	Weight.	

EXAMPLE II.

Suppose a certain Quantity of Water weighed 1 lb. 4 oz. 6.38 dr. in Avoirdupois Weight, how many cubic Inches are contained in that Water?

This is the Reverse of the former, therefore the Proportion is,

ON A.	ON B.	ON A.	ON B.	
As .578697 :	1 ::	20.399 :	35.25 =	Answer.
Tab. Numb.	Unity.	Weight.	Solid Inches.	

EXAMPLE III.

How much will a Bushel of Wheat of the best Sort weigh in Avoirdupois Weight?

PROPORTION.

ON B.	ON A.	ON B.	ON A.	lb. oz. dr.
As 1728 :	48.5 ::	2150.42 :	60.356 =	60 : 5 : 11
Sol. Inc.	Wt. of Sol. Incs.	Weight of		
in a Ft.	a Ft.	in a Bush.	a Bushel.	

In like Manner may the Weight of any other Body mentioned in the Table of specific Gravity be found; or on the contrary, by knowing the Weight you may find the Solidity of that Body.

C H A P. XV.

Of moneying of the Charges.

IN this Chapter is shewn the proper Method of moneying the Charges, or finding of the Amount of any Quantity of Goods when the Books are added up, and also the Method of reducing of one Sort of Goods to its equivalent Value in that of another Kind; in which are contained several useful and necessary Tables.

A R T. I.

To find the Duty of any Number of Barrels of Victuallers X Beer at 5 s. per Gallon.

R U L E.

THIS may be performed several Ways: But the readiest and quickest Method of doing it is by the taking of the aliquot Parts, because the Rate per Barrel is the aliquot Part of a Pound. Therefore divide the given Number of Barrels by 4, and the Quotient will be Pounds; to which add the Value of the Quarters, if any, and that Sum will be equal to the Duty of the Whole.

Or it may be found by the Cash Table in Pages 164 and 165.

E X A M P L E.

What is the Duty of $325\frac{1}{2}$ Barrels of Victuallers X Beer at 5 s. per Barrel.

O P E R A T I O N.

By Practice.	By Cash Table.
4(325=Given Num. Bar.	l. s. d.
of Barrels. 300 =	75 : 0 : 0
81 : 5	20 = 5 : 0 : 0
$\frac{1}{2}$ Bar. = 2 : 6	5 = 1 : 5 : 0
	$\frac{1}{2}$ = 2 : 6
Du.=81 : 7 : 6	$325\frac{1}{2}$ = 81 : 7 : 6

A R T. II.

To find the Duty of any Number of Barrels of Victuallers Small Beer at 1 s. 4 d. per Barrel.

R U L E.

TO the given Number of Barrels add one Third of that Sum, and, if there be any Quarters, add proportionably for them: The Sum of the Whole will be equal to the Duty in Shillings and Pence,

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Pence, which reduce into Pounds, and it is done.
Or it may be found by the Cash Table in Pages
164 and 165.

EXAMPLE.

What will the Duty of 295 $\frac{1}{2}$ Barrels of Vic-
tuallers Small Beer amount to at 1 s. 4 d. per
Barrel.

OPERATION.

By Practice.	By the Cash Tables.
3)295 Giv. Num. of Bs.	Bar. l. s. d.
98:4 $\frac{1}{2}$ of giv. Num.	200 = 12: 6: 8
0:8 $\frac{1}{2}$ Barrel.	90 = 6: 0: 0
	5 = 0: 6: 8
2)0)394:0 Shillings.	$\frac{1}{2}$ = 0: 0: 8
<u>£.19:14:0 = Answer.</u>	<u>295 $\frac{1}{2}$ = 19: 14: 0</u>

And in this Manner may the Amount of the
Duty of any other Sorts of Goods be found, (if
there be no fractional Parts in the Price of the In-
teger) whether they be Pounds, Yards, &c. either
by Practice, or by the Cash Tables in Pages 164
and 165; which is so easy that it needs no more
Examples.

A R T. III.

*To find the Duty of any Number of Barrels
of Common Brewers X Beer at 5 s. per
Barrel, with the Allowance of 2 in
every 23 Barrels.*

BY reason of this Allowance of 2 $\frac{1}{2}$ in every 23
Barrels there will be a Fraction in the Price of
the Integer; so that the Duty cannot be found by
taking of the aliquot Parts, as in the two last Ar-
ticles. But it may be found by the Cash Tables
for Common Brewers X Beer in Page 160 or 161,
or by a Factor found for that Purpose; which Fac-
tor is found in the following Manner:

The Duty of 23 Barrels at 5 s. per Barr. = 5:15:0
Allowance out of 23 Ba. is 2 $\frac{1}{2}$, and Duty = 0:12:6

Duty of 23 Barrels of C. Brewers X — = 5: 26

Then the Proportion for the Factor is

Barr. L. Barr. Decim.
As 23 : 5.125 :: 1 : .22826 = Factor.

Now having found the above Factor for C.
Brewers X Beer, if any Number of Barrels, and
Quarters of a Barrel reduced to a Decimal be mul-
tiplied by it, the Product will be equal to the Duty
in Pounds and Decimal Parts of a Pound.

EXAMPLE.

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EXAMPLE.

What is the Duty of $350 \frac{1}{2}$ Barrels of Common Brewers X Beer at 5 s. per Barrel with the Allowance of $2 \frac{1}{2}$ Barrels in 23.

OPERATION.

$222826 =$ Factor for X.

$350.5 =$ Given Number of Barrels.

1114130

1114130

668478

I. s. d.

$78.1005130 = 78 : 2 : 0 =$ the Duty.

ART. IV.

To find the Duty of any Number of Common Brewers Small Beer at 1 s. 4 d. per Barrel, the Allowance being $2 \frac{1}{2}$ in every 23 Barrels.

BY Reason of this Allowance there will be also a Fraction in the Price of one Barrel; so there must be a Factor found for Common Brewers Small Beer, as well as for the Strong. The Duty may be found by the Cash Table for C. Brewers Small Beer in Pages 162 and 163.

The Factor is found in the following Manner :

The Duty of 23 Barrels at 1 s. 4 d. per } I. s. d.
Barrel } $= 1 : 10 : 8$

The Allowance of $2 \frac{1}{2}$ Barrels } $= 0 : 3 : 4$

The Duty of 23 Barrels of C. Brewers } $= 1 : 7 : 4$
Small Beer }

Then the Proportion for the Factor is

Barr. L. Barr. Decimal of L.
As 23 : 1.366 :: 1 : .09942 = Factor.

Now having found the above Factor for Common Brewers Small Beer, if any Number of Barrels, and Quarters of a Barrel reduced to a Decimal, be multiplied by it, the Product will be equal to the Duty in Pounds and decimal Parts of a Pound.

EXAMPLE.

What will the Duty of $345 \frac{1}{4}$ Barrels of Common Brewers Small Beer amount to at 1 s. 4 d. per Barrel?

OPERATION.

$345.75 =$ Given Number of Barrels.

$.09942 =$ Factor for Small Beer.

69150

138300

311175

172875

I. s. d.

$20.5444650 = 20 : 10 : 10 \frac{1}{2} \frac{1}{8} =$ Duty.

A.C.

*A Cash Table for C. Brewers X at Five
of 2 $\frac{1}{2}$ in every*

No. of Bar.	0				$\frac{1}{2}$				$\frac{1}{2}$				$\frac{1}{2}$			
	L.	s.	d.	23 rd	L.	s.	d.	23 rd	L.	s.	d.	23 rd	L.	s.	d.	23 rd
0	0	0	0		0	1	1 $\frac{1}{2}$	11	0	2	2 $\frac{1}{2}$	22	0	3	4	10
1	0	4	5 $\frac{1}{2}$	21	0	5	6 $\frac{1}{2}$	9	0	6	8	20	0	7	9 $\frac{1}{2}$	8
2	0	8	10 $\frac{1}{2}$	19	0	10	0 $\frac{1}{2}$	7	0	11	1 $\frac{1}{2}$	18	0	12	3	6
3	0	13	4 $\frac{1}{2}$	17	0	14	5 $\frac{1}{2}$	5	0	15	7	16	0	16	8 $\frac{1}{2}$	4
4	0	17	9 $\frac{1}{2}$	15	0	18	11 $\frac{1}{2}$	3	1	0	0 $\frac{1}{2}$	14	1	1	2	2
5	1	2	3 $\frac{1}{2}$	13	1	3	4 $\frac{1}{2}$	1	1	4	6	12	1	5	7 $\frac{1}{2}$	
6	1	6	8 $\frac{1}{2}$	11	1	7	10	22	1	8	11 $\frac{1}{2}$	10	1	10	0 $\frac{1}{2}$	21
7	1	11	2 $\frac{1}{2}$	9	1	12	3 $\frac{1}{2}$	20	1	13	5	8	1	14	6 $\frac{1}{2}$	19
8	1	15	7 $\frac{1}{2}$	7	1	16	9	18	1	17	10 $\frac{1}{2}$	6	1	18	11 $\frac{1}{2}$	17
9	2	0	1 $\frac{1}{2}$	5	2	1	2 $\frac{1}{2}$	16	2	2	4	4	2	3	5 $\frac{1}{2}$	15
10	2	4	6 $\frac{1}{2}$	3	2	5	8	14	2	6	9 $\frac{1}{2}$	2	2	7	10 $\frac{1}{2}$	13
11	2	9	0 $\frac{1}{2}$	1	2	10	1 $\frac{1}{2}$	12	2	11	3		2	12	4 $\frac{1}{2}$	11
12	2	13	5 $\frac{1}{2}$	22	2	14	7	10	2	15	8 $\frac{1}{2}$	21	2	16	9 $\frac{1}{2}$	9
13	2	17	11	20	2	19	0 $\frac{1}{2}$	8	3	0	1 $\frac{1}{2}$	19	3	1	3 $\frac{1}{2}$	7
14	3	2	4 $\frac{1}{2}$	18	3	3	6	6	3	4	7 $\frac{1}{2}$	17	3	5	8 $\frac{1}{2}$	5
15	3	6	10	16	3	7	11 $\frac{1}{2}$	4	3	9	0 $\frac{1}{2}$	15	3	10	2 $\frac{1}{2}$	3
16	3	11	3 $\frac{1}{2}$	14	3	12	5	2	3	13	6 $\frac{1}{2}$	13	3	14	7 $\frac{1}{2}$	1
17	3	15	9	12	3	16	10 $\frac{1}{2}$		3	17	11 $\frac{1}{2}$	11	3	19	1	22
18	4	0	2 $\frac{1}{2}$	10	4	1	3 $\frac{1}{2}$	21	4	2	5 $\frac{1}{2}$	9	4	3	6 $\frac{1}{2}$	20
19	4	4	8	8	4	5	9 $\frac{1}{2}$	19	4	6	10 $\frac{1}{2}$	7	4	8	0	18
20	4	9	1 $\frac{1}{2}$	6	4	10	2 $\frac{1}{2}$	17	4	11	4 $\frac{1}{2}$	5	4	12	5 $\frac{1}{2}$	16
21	4	13	7	4	4	14	8 $\frac{1}{2}$	15	4	15	9 $\frac{1}{2}$	3	4	16	11	14
22	4	18	0 $\frac{1}{2}$	2	4	19	1 $\frac{1}{2}$	13	5	0	3 $\frac{1}{2}$	1	5	1	4 $\frac{1}{2}$	12

The Use of the Cash Table for C. Brewers X Beer :

1. If the Number of Barrels and Firkins are under 23 seek in the first Column above for the Number of Barrels, and for the Firkins, if any; at the Head of the Table, and at the Angle of Meeting, you have the Duty in Pounds, Shillings, Pence, Farthings, and 23d Parts of a Farthing.

2. If the given Number of Barrels exceed 23, then seek for the nearest less Number of even Barrels on the next Page, and set them down with their Duty; then subtract this nearest less Number of Barrels from the given Number of Barrels, and with the Remainder seek above for their Duty as in the first Case, which add to the former, and their Sum will be equal to the Duty of the Whole.

E X A M P L E.

What is the Duty of 350 $\frac{1}{2}$ Barrels of C. Brewers X Beer.

O P E R A T I O N.

Given Number of Barrels. = 350 $\frac{1}{2}$

The nearest less ——— = 345 Duty = 76:17:6

The Remainder ——— = 5 $\frac{1}{2}$ Duty = 1:4:6 $\frac{1}{2}$

The Duty of the Whole ——— = 78: 2:0 $\frac{1}{2}$

Shillings

Shillings per Barrel, with the Allowance
23 Barrels.

N ^o . of Barrels.	L.	s.	d.	N ^o . of Barrels.	L.	s.	d.
23	5	2	6	1081	240	17	6
46	10	5	0	1104	246	0	6
69	15	7	6	1127	252	2	6
92	20	10	0	1150	258	5	6
115	25	12	6	1173	264	7	6
138	30	15	0	1196	269	10	0
161	35	17	6	1219	275	12	6
184	40	0	0	1242	281	15	0
207	45	2	6	1265	287	17	6
230	50	5	0	1288	293	0	0
253	55	7	6	1311	299	2	6
276	60	10	0	1334	305	5	0
299	65	12	6	1357	311	7	6
322	70	15	0	1380	317	10	0
345	75	17	6	1403	323	12	6
368	80	0	0	1426	329	15	0
391	85	2	6	1449	335	17	6
414	90	5	0	1472	341	0	0
437	95	7	6	1495	347	2	6
460	100	10	0	1518	353	5	0
483	105	12	6	1541	359	7	6
506	110	15	0	1564	365	10	0
529	115	17	6	1587	371	12	6
552	120	0	0	1610	377	15	0
575	125	2	6	1633	383	17	6
598	130	5	0	1656	389	0	0
621	135	7	6	1679	395	2	6
644	140	10	0	1702	401	5	0
667	145	12	6	1725	407	7	6
690	150	15	0	1748	413	10	0
713	155	17	6	1771	419	12	6
736	160	0	0	1794	425	15	0
759	165	2	6	1817	431	17	6
782	170	5	0	1840	437	0	0
805	175	7	6	1863	443	2	6
828	180	10	0	1886	449	5	0
851	185	12	6	1909	455	7	6
874	190	15	0	1932	461	10	0
897	195	17	6	1955	467	12	6
920	200	0	0	1978	473	15	0
943	205	2	6	2001	479	17	6
966	210	5	0	2024	485	0	0
989	215	7	6	2047	491	2	6
1012	220	10	0	2070	497	5	0
1035	225	12	6	2093	503	7	6
1058	230	15	0	2116	509	10	0

M

A Cope

*A Cash Table for Common Brewers Small
allowance of 2 $\frac{1}{2}$ Barrels*

N ^o . of Barrels	0				$\frac{1}{2}$				$\frac{1}{2}$				$\frac{1}{2}$			
	L.	s.	d.	23 ^d	L.	s.	d.	23 ^d	L.	s.	d.	23 ^d	L.	s.	d.	23 ^d
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	1	0	1	5	7	0	1	9	13	0	2	0	18
2	0	2	4	2	0	2	6	8	0	2	11	14	0	3	3	19
3	0	3	6	3	0	3	10	9	0	4	12	15	0	4	5	20
4	0	4	9	4	0	4	0	10	0	5	4	16	0	5	7	21
5	0	5	11	5	0	5	2	11	0	6	0	17	0	6	10	22
6	0	6	13	6	0	6	5	12	0	7	3	18	0	7	13	23
7	0	7	15	7	0	7	8	13	0	8	6	19	0	8	16	24
8	0	8	17	8	0	8	11	14	0	9	9	20	0	9	19	25
9	0	9	19	9	0	9	14	15	0	10	12	21	0	10	22	26
10	0	10	21	10	0	10	17	16	0	11	15	22	0	11	25	27
11	0	11	23	11	0	11	20	17	0	12	18	23	0	12	28	28
12	0	12	25	12	0	12	23	18	0	13	21	24	0	13	31	29
13	0	13	27	13	0	13	26	19	0	14	24	25	0	14	34	30
14	0	14	29	14	0	14	29	20	0	15	27	26	0	15	37	31
15	0	15	31	15	0	15	32	21	0	16	30	27	0	16	40	32
16	0	16	33	16	0	16	35	22	0	17	33	28	0	17	43	33
17	0	17	35	17	0	17	38	23	0	18	36	29	0	18	46	34
18	0	18	37	18	0	18	41	24	0	19	39	30	0	19	49	35
19	0	19	39	19	0	19	44	25	0	20	42	31	0	20	52	36
20	0	20	41	20	0	20	47	26	0	21	45	32	0	21	55	37
21	0	21	43	21	0	21	50	27	0	22	48	33	0	22	58	38
22	0	22	45	22	0	22	53	28	0	23	51	34	0	23	61	39

The Use of the Cash Table for C. Brewer's Small Beer.

1. If the Number of Barrels and Firkins are under 23, look in the first Column above for the Number of Barrels, and for the Firkins (if there be any) at the Top of this Side; then at the Angle of meeting you will find the exact Duty in Pounds, Shillings and Pence, Farthings and 23d Parts of a Farthing.

2. If the given Number of Barrels exceed 23, then seek for the nearest less Number of even Barrels on the next Page, and set them down with their Duty; then subtract the nearest less Number of Barrels, &c. from the given Number of Barrels, &c. and with the Remainder (if any) seek above for their Duty, as in the 1st Case; which add to the former, and the Sum will be equal to the Duty of the Whole.

E X A M P L E.

What is the Duty of 345 $\frac{1}{2}$ Barrels of C. Brewers Small Beer?

O P E R A T I O N.

Given N^o. of Barrels = 345 $\frac{1}{2}$

Next less N^o. of ditto = 345 Duty = 20:10:0

Remainder — = $\frac{1}{2}$ Duty = 0:0:10 $\frac{1}{2}$

The whole Duty — = 20:10:10 $\frac{1}{2}$

Beer

Beer 1 s. 4 d. per Barrel, with the A-
in every 23 Barrels.

N ^o . of Barrel.	L.	s.	d.	N ^o . of Barrel.	L.	s.	d.
23	1	7	4	1081	64	4	8
46	2	14	8	1104	65	12	0
69	4	2	0	1127	66	19	4
92	5	9	4	1150	68	6	8
115	6	16	8	1173	69	14	0
138	8	4	0	1196	71	1	4
161	9	11	4	1219	72	8	8
184	10	18	8	1242	73	16	0
207	12	6	0	1265	75	3	4
230	13	13	4	1288	76	10	8
253	15	0	8	1311	77	18	0
276	16	8	0	1334	79	5	4
299	17	15	4	1357	80	12	8
322	19	2	8	1380	82	0	0
345	20	10	0	1403	83	7	4
368	21	17	4	1426	84	14	8
391	23	4	8	1449	86	2	0
414	24	12	0	1472	87	9	4
437	25	19	4	1495	88	16	8
460	27	6	8	1518	90	4	0
483	28	14	0	1541	91	11	4
506	30	1	4	1564	92	18	8
529	31	8	8	1587	94	6	0
552	32	16	0	1610	95	13	4
575	34	3	4	1633	97	0	8
598	35	10	8	1656	98	8	0
621	36	18	0	1679	99	15	4
644	38	5	4	1702	101	2	8
667	39	12	8	1725	102	10	0
690	41	0	0	1748	103	17	4
713	42	7	4	1771	104	4	8
736	43	14	8	1794	106	12	0
759	45	2	0	1817	107	19	4
782	46	9	4	1840	109	6	8
805	47	16	8	1863	110	14	0
828	49	4	0	1886	112	1	4
851	50	11	4	1909	113	8	8
874	51	18	8	1932	114	16	0
897	53	6	0	1955	116	3	4
920	54	13	4	1978	117	10	8
943	56	0	8	2001	118	18	0
966	57	8	0	2024	120	5	4
989	58	15	4	2047	121	12	8
1012	60	2	8	2070	123	0	0
1035	61	10	0	2093	124	7	4
1058	62	17	4	2116	125	14	8

A Cash Table for Tanners, Tawers, Chandelers

Numb. of Pds. Brs. Bushels.	At 1d.			At 2d.			At 3d.			At 4d.		
	L.	s.	d.	L.	s.	d.	L.	s.	d.	L.	s.	d.
1	0	0	1	0	0	2	0	0	3	0	0	4
2	0	0	2	0	0	4	0	0	6	0	0	8
3	0	0	3	0	0	6	0	0	9	0	0	12
4	0	0	4	0	0	8	0	0	12	0	0	16
5	0	0	5	0	0	10	0	0	15	0	0	20
6	0	0	6	0	0	12	0	0	18	0	0	24
7	0	0	7	0	0	14	0	0	21	0	0	28
8	0	0	8	0	0	16	0	0	24	0	0	32
9	0	0	9	0	0	18	0	0	27	0	0	36
10	0	0	10	0	0	20	0	0	30	0	0	40
11	0	0	11	0	0	22	0	0	33	0	0	44
12	0	0	12	0	0	24	0	0	36	0	0	48
13	0	0	13	0	0	26	0	0	39	0	0	52
14	0	0	14	0	0	28	0	0	42	0	0	56
15	0	0	15	0	0	30	0	0	45	0	0	60
16	0	0	16	0	0	32	0	0	48	0	0	64
17	0	0	17	0	0	34	0	0	51	0	0	68
18	0	0	18	0	0	36	0	0	54	0	0	72
19	0	0	19	0	0	38	0	0	57	0	0	76
20	0	0	20	0	0	40	0	0	60	0	0	80
30	0	2	6	0	3	10	0	5	0	0	7	6
40	0	3	4	0	4	8	0	6	8	0	10	0
50	0	4	2	0	5	6	0	8	4	0	12	6
60	0	5	0	0	6	4	0	10	2	0	15	0
70	0	5	10	0	7	2	0	11	0	0	17	6
80	0	6	8	0	8	0	0	12	4	0	18	0
90	0	7	6	0	9	4	0	13	2	0	19	6
100	0	8	4	0	10	2	0	14	0	0	21	0
200	0	16	8	0	20	0	0	28	0	0	36	0
300	0	24	0	0	30	0	0	42	0	0	54	0
400	0	32	0	0	40	0	0	56	0	0	72	0
500	0	40	0	0	50	0	0	70	0	0	90	0
600	0	48	0	0	60	0	0	84	0	0	108	0
700	0	56	0	0	70	0	0	98	0	0	126	0
800	0	64	0	0	80	0	0	112	0	0	144	0
900	0	72	0	0	90	0	0	126	0	0	162	0
1000	0	80	0	0	100	0	0	140	0	0	180	0
2000	0	16	8	0	20	0	0	28	0	0	36	0
3000	0	24	0	0	30	0	0	42	0	0	54	0
4000	0	32	0	0	40	0	0	56	0	0	72	0
5000	0	40	0	0	50	0	0	70	0	0	90	0
6000	0	48	0	0	60	0	0	84	0	0	108	0
7000	0	56	0	0	70	0	0	98	0	0	126	0
8000	0	64	0	0	80	0	0	112	0	0	144	0
9000	0	72	0	0	90	0	0	126	0	0	162	0
10000	0	80	0	0	100	0	0	140	0	0	180	0

Paper-Makers, Maltsters, Victuallers, &c.

At 4 $\frac{1}{2}$. 2 d.	At 5d.	At 11. 4d	At 4s.	At 5s.	At 6s. 2d.
L. s. d. 10 ^m	L. s. d.	L. s. d.	L. s.	L. s.	L. s. d.
0 0 4 $\frac{1}{2}$ 2	0 0 6	0 1 4	0 4	0 5	0 6 8
0 0 9 $\frac{1}{2}$ 4	0 1 0	0 2 8	0 8	0 10	0 13 4
0 1 2 $\frac{1}{2}$ 6	0 1 6	0 4 0	0 12	0 15	1 0 0
0 1 7 $\frac{1}{2}$ 8	0 2 0	0 5 4	0 16	1 0	1 4 8
0 2 0	0 2 6	0 6 8	1 0	1 5	1 13 4
0 2 4 $\frac{1}{2}$ 2	0 3 0	0 8 0	1 4	1 10	2 0 0
0 2 9 $\frac{1}{2}$ 4	0 3 6	0 9 4	1 8	1 15	2 6 8
0 3 2 $\frac{1}{2}$ 6	0 4 0	0 10 8	1 12	2 0	2 13 4
0 3 7 $\frac{1}{2}$ 8	0 4 6	0 12 0	1 16	2 5	3 0 0
0 4 0	0 5 0	0 13 4	2 0	2 10	3 6 8
0 4 4 $\frac{1}{2}$ 2	0 5 6	0 14 8	2 4	2 15	3 13 4
0 4 9 $\frac{1}{2}$ 4	0 6 0	0 16 0	2 8	3 0	4 0 0
0 5 2 $\frac{1}{2}$ 6	0 6 6	0 17 4	2 12	3 5	4 6 8
0 5 7 $\frac{1}{2}$ 8	0 7 0	0 18 8	2 16	3 10	4 13 4
0 6 0	0 7 6	1 0 0	3 0	3 15	5 0 0
0 6 4 $\frac{1}{2}$ 2	0 8 0	1 1 4	3 4	4 0	5 6 8
0 6 9 $\frac{1}{2}$ 4	0 8 6	1 2 8	3 8	4 5	5 13 4
0 7 2 $\frac{1}{2}$ 6	0 9 0	1 4 0	3 12	4 10	6 0 0
0 7 7 $\frac{1}{2}$ 8	0 9 6	1 5 4	3 16	4 15	6 6 8
0 8 0	0 10 0	1 6 8	4 0	5 0	6 13 4
0 12 0	0 15 0	2 0 0	6 0	7 10	10 0 0
0 16 0	1 0 0	2 13 4	8 0	10 0	13 6 8
1 0 0	1 5 0	3 6 8	10 0	12 10	16 13 4
1 4 0	1 10 0	4 0 0	12 0	15 0	20 0 0
1 8 0	1 15 0	4 13 4	14 0	17 10	23 6 8
1 12 0	2 0 0	5 6 8	16 0	20 0	26 13 4
1 16 0	2 5 0	6 0 0	18 0	22 10	30 0 0
2 0 0	2 10 0	6 13 4	20 0	25 0	33 6 8
4 0 0	5 0 0	13 6 8	40 0	50 0	66 13 4
6 0 0	7 10 0	20 0 0	60 0	75 0	100 0 0
8 0 0	10 0 0	26 13 4	80 0	100 0	133 6 8
10 0 0	12 10 0	33 6 8	100 0	125 0	166 13 4
12 0 0	15 0 0	40 0 0	120 0	150 0	200 0 0
14 0 0	17 10 0	46 13 4	140 0	175 0	233 6 8
16 0 0	20 0 0	53 6 8	160 0	200 0	266 13 4
18 0 0	22 10 0	60 0 0	180 0	225 0	300 0 0
20 0 0	25 0 0	66 13 4	200 0	250 0	333 6 8
40 0 0	50 0 0	133 6 8	400 0	500 0	666 13 4
60 0 0	75 0 0	200 0 0	600 0	750 0	1000 0 0
80 0 0	100 0 0	266 13 4	800 0	1000 0	1333 6 8
100 0 0	125 0 0	333 6 8	1000 0	1250 0	1666 13 4
120 0 0	150 0 0	400 0 0	1200 0	1500 0	2000 0 0
140 0 0	175 0 0	466 13 4	1400 0	1750 0	2333 6 8
160 0 0	200 0 0	533 6 8	1600 0	2000 0	2666 13 4
180 0 0	225 0 0	600 0 0	1800 0	2250 0	3000 0 0
200 0 0	250 0 0	666 13 4	2000 0	2500 0	3333 6 8

A R T. V.

To reduce the gross Bushels of Malt taken in the Cistern or Couch, and Floor, to neat Bushels.

THE Duty on every Winchester Bushel of Malt is 6d. and it is supposed that Barley after it is first wetted or steeped in the Cistern, and stood there its proper Time, and from thence emptied into the Couch, and lain there about 30 Hours, rises or increases to about $\frac{1}{3}$ Part more than it was before: Therefore 4 Bushels in every 20 are to be allowed for that Increase.

But when the Malt has been out of the Cistern above 30 Hours it is deemed to be a Floor of Malt, and it is supposed that a Bushel of dry Barley thus wetted and steeped, &c. and afterwards thrown out into the Floor, and there grown according to the usual Custom, will increase or rise to two Bushels, or double to what it was before: Therefore 10 Bushels in every 20 are to be allowed for that Increase.

Now in order to find proper Factors to reduce each of these Bushels to their equivalent Value in neat Bushels, observe the following Method:

1. For the Cistern or Couch Bushels.

From 20 = Bushels.

Subtract 4 = $\frac{1}{3}$

Remains 16 = $\frac{2}{3}$ = ,8 = Factor for Cistern or Couch Bushels.

If any Number of Bushels from Cistern or Couch be multiplied by the above Factor the Product will be equal to the neat Bushels at 6d. per Bushel. As for Example, In 100 Bushels from Cistern or Couch how many neat Bushels?

O P E R A T I O N.

100 = Bushels.

,8 = Factor.

80,0 = Neat Bushels.

2. For the Floor Bushels.

There being 10 Bushels in every 20 to be allowed for the Increase of Floor Bushels, therefore the
Floor

Of reducing of Malt Bushels, &c. 167

Floor Bushel is $= \frac{12}{25} = \frac{1}{2} = .5$, a common Factor for Floor Bushels.

If any Number of Floor Bushels be multiplied by the Factor above found, the Product will be equal to the neat Bushels at 6 d. each Bushel. As for Example. In 160 Bushels from the Floor how many neat Bushels?

OPERATION.

160 = Bushels.

.5 = Factor.

80,0 = Neat Bushels.

A R T. VI.

To find Factors for reducing of Couch Bushels to Floor Bushels, and on the contrary for reducing Floor Bushels to Couch Bushels, in order to find from whence the Charge will arise.

R U L E.

THE Factors found in the last Article are to each other as Unity to the required Factors: Therefore the Proportions are as follow:

1. As .5 : .8 :: 1 : 1,6 = Factor for Couch Bushels.

2. As .8 : .5 :: 1 : .625 = Factor for Floor Bushels.

Now by these Factors it will be easy to find whether the Charge will arise from the Best of the Cistern and Couch, or from the Floor; for if the Bushels from the Best of the Cistern and Couch multiplied by 1,6, the Factor above found, produce more Bushels than the Floor, the Charge will be from the Best: But if it produces less, then the Charge will be from the Floor.

Or the Charge may be found by multiplying the Floor Bushels by .625; and if the Product be more than the Bushels from Best, the Charge will be from the Floor; but if less then from the Best of the Cistern and Couch.

E X A M P L E.

Suppose the Content of a Floor Gage of Malt = 161 Bushels, and the Content of the Best, Cistern, or Couch were = 100 Bushels, from which will the Charge arise?

168 Of reducing of Malt Bushels.

The Proportion by Pen or Rule.

$$\begin{array}{ccccccc} \text{on B.} & \text{on A.} & & \text{on B.} & \text{on A.} & & \\ \text{As } 1 & : 1,6 & :: & 100 & : 160 = & \text{Bushels.} & \\ \text{Unity. Factor.} & & & \text{Couch.} & \text{Floor.} & & \end{array}$$

By which I find the Amount of the Couch, &c. is = 160 Floor Bushels, that is one Bushel less than the Number of Floor Bushels: Therefore the Charge will arise from the Floor.

Or the same may be found by this Proportion.

$$\begin{array}{ccccccc} \text{on B.} & \text{on A.} & & \text{on B.} & \text{on A.} & & \\ \text{As } 1 & : ,625 & :: & 160 & : 100,6 = & \text{Bushels.} & \\ \text{Unity. Factor.} & & & \text{Floor.} & \text{Couch.} & & \end{array}$$

From whence it also appears that the Floor Gage is the best, viz. it amounts to the most Duty.

A R T. VII.

To find the Duty of any Number of Bushels from the Best of the Cistern or Couch.

R U L E.

THIS may be done by the Cash Table in Pages 164 and 165; or by a Factor, which is found in the following Manner.

The Duty of 20 Bushels of Malt from Cistern or Couch, with the Allowance of 4 in 20, or the $\frac{1}{5}$ Part, is 8 s. which reduced to the Decimal of a Pound Sterling will = ,4, and then the Proportion for the Factor will be

$$\begin{array}{ccccccc} \text{Bushels.} & \text{Duty.} & \text{Bushels.} & \text{Duty.} & & & \\ \text{As } 20 & : ,4 & :: & 1 & : ,02 = & \text{the Factor.} & \end{array}$$

Now if any Number of Bushels from Cistern or Couch be multiplied by the above Factor, the Product will be equal to the Duty in Pounds and decimal Parts of a Pound.

E X A M P L E.

How much will the Duty of 2055 Bushels of Malt from the Best of the Cistern or Couch amount to at 4 d. 3 f. 2 Tenths per Bushel?

O P E R A T I O N.

By the Cash Table.

By the Factor.

$$\begin{array}{rcl} & \text{l. s. d.} & 2055 = \text{N}^{\circ} \text{ of Bushels.} \\ 2000 = \text{Bushels} & = 40:0:0 & ,02 = \text{Factor.} \\ 50 = \text{Bushels} & = 1:0:0 & \\ 5 = \text{Ditto} & = 0:2:0 & 41,10 \\ \hline 2055 = \text{Duty} & = 41:2:0 & 20 \\ & & \text{l. s. d.} \\ & & 412,00 = 41:2:0 \\ & & \text{A R T.} \end{array}$$

Of finding the Charge of Floor Bushels. 169

A R T. VIII.

To find the Duty of any Number of Bushels from the Floor.

R U L E.

THIS may be done by the Cash Table in Page 164 and 165, or by dividing the given Number of Bushels by 4, and the Quotient will be Shillings, which reduced into Pounds will be equal to the Duty required.

E X A M P L E.

What is the Duty of 2560 Bushels of Malt from the Floor at 3 d. per Bushels.

O P E R A T I O N.

By the Cash Table	By Practice.
l. s. d.	4)2560=N ^o . of Bs.
2000 Bushels = 25: 0:0	—
500 Ditto = 6: 5:0	250)640=Shillings.
60 Ditto = 0:15:0	—
—	£ 32 =Duty.
2560 Duty = 32: 0:0	

In like Manner may the Duty of any other Sort of Goods be found, either by the Cash Table or by Practice; which being so easy I shall add no more Examples, but proceed to the next Article.

A R T. IX.

To find Factors for reducing the odd Gallons of one Denomination to their equivalent Value in those of another Denomination, so as they may produce the same Duty.

R U L E.

- I**F the odd Gallons to be reduced only differ in Duty, and there be the same Number of Gallons of each Sort in a Barrel, Hoghead, &c. then the Proportion will be as the Duty of a Barrel or Hoghead, &c. of that which is to be reduced is to Unity, so is the Duty of that which it is to be reduced to the Factor.
- If they not only differ in Duty, but also in the Number of Gallons to a Barrel, Hoghead, &c. then by the Rule of Three Direct find the Duty of one Gallon of each, and the former Proportion will hold good, viz. As the Duty of one Gallon of that to be reduced is to Unity, so is the Duty of the other to the Factor required.

E X A M P L E.

EXAMPLE.

To find a Factor to reduce X to VI Beer.

OPERATION.

Pence. Unity. Pence. Gallons.

As 60 : 1 :: 16 : 3.75 = Factor.

$$\begin{array}{r}
 1 \\
 \hline
 16 \overline{) 60} 3.75 = \text{Factor for X to VI.} \\
 \underline{48} \\
 120 \\
 \underline{112} \\
 .80 \\
 80 \\
 \hline
 \text{Rem. } 0
 \end{array}$$

This is Indirect Proportion, because less Duty requires more Gallons : Therefore I multiply the first and second Terms together and divide by the third.

EXAMPLE II.

To find a Factor to reduce VI Beer to X Beer.

OPERATION.

Pence. Unity. Pence. Gallons.

As 16 : 1 :: 60 : .266 + = Factor.

$$\begin{array}{r}
 1 \\
 \hline
 60 \overline{) 16.0} .266 + = \text{Factor for VI to X.} \\
 \underline{120} \\
 400 \\
 \underline{360} \\
 400 \\
 \underline{360} \\
 \text{Rem. } .40
 \end{array}$$

EXAMPLE III.

To find a Factor to reduce Cyder to X Beer. In this Example the Integers differ as well as the Duty : Therefore by Case 2^d find the Duty of a Gallon of each.

1. For the Duty of 1 Gallon of Cyder.

The Proportion is

Galls. Duty. Galls. Duty.

As 63 : 10s. 8d. :: 1 : 2.0317 = Duty of 1 Gall. of Cyder.

2. For the Duty of 1 Gallon of X Beer.

The Proportion is

Gall. Pence. Gall. Pence.

As 34 : 60 :: 1 : 1.7647 = Duty of 1 Gall. of X Beer.

Then

for reducing of odd Gallons, &c. 171.

Then for the Factor say by Indirect Proportion,

Duty of Cdr. Gall. Duty of X. Factor.

As 2.0317 : 1 :: 1.7647 : 1.15 = for Cyder to X.

E X A M P L E IV.

To find a Factor to reduce Cyder to *vi* Beer.

In this Example the Gallon differs also, as well as the Duty : Therefore by Case the 2d. find the Duty of a Gallon of each.

The Proportion for the Duty of a Gallon of VI is

Gall. Pence. Gall. Pence.

As 34 : 16 :: 1 : .4706 = Duty of a Gall. of VI.

The Duty of 1 Gallon of Cyder was found before to be = 2.0317 d. so the Proportion for the Factor is,

Duty of C. Gall. Duty of VI. Factor.

As 2.0317 : 1 :: .4706 : 4.32 = for Cyder to VI.

In like Manner may any other Factor be found for reducing of odd Gallons of one Denomination to their equivalent Value in those of another Denomination ; and in this Manner were the Factors found in the following Table.

A Table of Factors.

Denominations.	Factors.
Strong Beer to Small	3.75
Small Beer to Strong	.266
Cyder to Strong Beer	1.15
Cyder to Small Beer	4.32
Verjuice to Strong Beer	.719
Vinegar to Strong Beer	1.75
Sweets to Strong Beer	2.59
Sweets to Small Beer	9.73
Mum to Strong Beer	5.312
Cyder or Perry to Couch Bushels	.15873
Cyder or Perry to Floor Bushels — —	.254

Or

Of the Use of the foregoing Table of Factors.

In making up of the Accompts only the even Quarters of a Barrel or Hoghead are charged, and the odd Gallons (if any) are carried forward to the next Accompts: But it frequently happens that a Trader may not have any more of that Kind of Goods to be charged for a long while, and perhaps not at all; therefore it will be necessary to reduce the odd Gallons which remain to any other Sort of Goods that are still depending, or to a Fourth of the least Denomination, which may be done by the Help of the foregoing Table.

For Example, suppose it were required to reduce 4.5 Gallons of X to VI, how many Gallons of VI would it produce?

OPERATION.

3.75 = Factor for X to VI.

4.5 = Gallons of X.

1875

1500

16.875 = Gallons of VI.

This produces 16.875 Gallons of VI, which may be reckoned 17 Gallons, or a Kilderkin of VI, the Duty of which is equal to the Duty of 4.5 of X Beer.

The like may be done by the odd Gallons of any other Denomination mentioned in the Table. But the Operation is best performed by the Lines a and b on the Rule, which is so easy that it requires no more Examples.

ART. X.

To find Factors for reducing Ale Measure to Wine and Corn Measure; and e. contra, Corn to Ale and Wine Measure.

TO do this observe the following Proportions, which are wrought by the Rule of Three Inverse.

PROPORTIONS.

As	{	282	: 12	231	: 11.280779	}	Factors for	{	A to W
		231	: 11	282	: 8.19148				W to A
		282	: 11	2150.42	: 13.1137				A to C
		2150.42	: 11	282	: 7.625602				C to A
		231	: 11	2150.42	: 10.7422				W to C
		2150.42	: 11	231	: 9.309177				C to W

Of reducing Ale to Wine Measure, &c. 273

Of the Use of the foregoing Factors.

If any Number of Gallons of Ale, Wine, or Corn Measure be multiplied by its proper Factor the Product will be the Number of Gallons reduced to the Measure desired.

EXAMPLE.

Reduce 63 Gallons of Wine Measure to Ale Measure.

OPERATION.

,819148 = Factor for Wine to Ale Meas.
63 = Given Numb. of W. Galls.

2457444
4914888

Prod. 51.606324 = N°. of Galls. in Ale Meas.

By this we see that $51 \frac{1}{2}$ Gallons of Ale Measure are nearly equal to a Wine Hogshead; and thus may any Number of Gallons of Ale Measure be reduced to Corn or Wine Measure by the Help of the foregoing Factors.

But for the sake of whole Numbers the following Proportion may serve well enough for reducing Ale to Wine; and *e contra*, Wine to Ale Measure, viz. as 9 is to 11, so is Ale to Wine Measure; and as 11 is to 9, so is Wine to Ale Measure.

For Example, In 51.6 Gallons of Ale Measure how many Gallons of Wine Measure?

Proportion by the Rule.

ON B. ON A. ON B. ON A.
As 9 : 11 :: 51.6 : 63 = Wine Gallons.

Proportion for Wine to Ale Measure.

As 11 : 9 :: 63 : 51.6 = Ale Gallons.

By this Proportion the Answer comes out nearly the same as that before found by the Factor for Wine to Ale Measure.

A R T.

ART. XI.

To find Factors for Salaries, both for common and Leap Years at any Rate per Annum.

RULE.

AS the Number of Days in a Year is to the Salary per Annum, so is one Day to its Salary; which Decimal of the Salary for one Day will be a proper Factor to find the Salary of any Number of Days at that Rate.

EXAMPLE.

Suppose the Salary to be 5 l. per Annum, what will the Factors be at that Rate, both for a common Year containing 365 Days, and a Leap Year containing 366 Days?

1. For a common Year.

Days. L. Day. Decimal of a L.

As 365 : 5 :: 1 : ,013699 = Fact. for com. Yr.

365)5.00(.013699

365

1350

1095

2550

2190

3600

3285

3150

3285

Rem. — 135

2. For the Leap Year.

Days. L. Day. Decimal.

As 366 : 5 :: 1 : ,013661 = Factor for Lp. Yr.

366)5.00(.013661

366

1340

1098

2420

2196

.2240

2196

.440

366

Rem. .74

Thus have I found Factors both for common and Leap Years at the Rate of 5 l. per Annum, and in this Manner was the following Table of Factors calculated.

At Table

A Table of Factors for Salaries.

Salaries per Annum.			Factor for a comm. Year.	Factor for a Leap Year.
L.	s.	d.		
0	0	0	13699	13661
10	0	0	127397	127322
1	0	0	14151	140983
20	0	0	105494	10464
25	0	0	1068493	106806
30	0	0	1082194	1081967
40	0	0	109589	109285
48	2	6	1131849	1131430
50	0	0	1136986	113661
52	0	0	1142465	1142076
60	0	0	1164383	1163934
70	0	0	1191780	1191251
80	0	0	1219178	1218579
86	12	6	1237329	1233948
90	0	0	1246575	1245903
100	0	0	1273072	1273224
110	10	0	1316439	1315574
120	0	0	1328766	1327869
200	0	0	1547974	1546448
300	0	0	1821917	1819672
400	0	0	1095948	1092856
500	0	0	1369863	1366119
600	0	0	1643834	1639344
700	0	0	1917808	1912568
800	0	0	2191896	2185792
900	0	0	2465811	2459616
1000	0	0	2739726	2732240

A Table for finding the Number of Days from any Day in one Month to the same Day in another Month.

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	365	334	306	273	241	210	181	152	123	94	65	36
Feb.	31	365	337	305	272	241	210	181	152	123	94	65
March	59	28	365	334	304	273	241	210	181	152	123	94
Jan.	365	334	306	273	241	210	181	152	123	94	65	36
Feb.	31	365	337	305	272	241	210	181	152	123	94	65
March	59	28	365	334	304	273	241	210	181	152	123	94
April	90	59	31	365	335	304	274	243	212	181	151	120
May	120	89	61	30	365	334	304	273	242	211	180	149
June	151	120	92	61	31	365	335	304	273	242	211	180
July	181	150	122	91	62	30	365	334	303	272	241	179
August	212	181	153	122	92	61	31	365	334	303	272	179
Sept.	243	212	184	153	123	92	62	32	365	335	304	273
Oct.	273	242	214	183	153	124	93	63	30	365	334	272
Nov.	304	273	245	214	184	154	125	94	64	31	365	335
Dec.	334	303	275	214	184	154	125	94	64	31	365	335

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in

Another Example.

Suppose the Salary 50 l. per Annum. as before, how much would be due from the 24th of June to the 30th of August following in a Leap Year?

First for the Number of Days.

OPERATION.

By the Table.	By adding the Days of each Month included between.
The N ^o . under June at the Side, and against August in the Top Column is ————	June 6
Time of ending Aug. 30	July 31
Beginning June 24	Aug. 30
Difference add = 6	67 = N ^o . of Days.
Number of Days = 67	

Then for the Salary.

Factor at 50 l. per Ann. for Leap Year =	136612
Number of Days =	67
	956284
Salary.	819672
l. s. d.	
9 : 3 : 0 $\frac{1}{4}$ fere =	9,153004

The Decimal is reduced by Inspection according to the Directions in Chap. i. Art. vii.

Note, If any of the Factors in the Table are reduced, they will shew the Amount of a Day's Salary at any Rate therein mentioned.

In the foregoing Examples only the gross Salary is found, whereas there must be 9 d. in a Pound deducted for Tax and Charity; therefore the neat Salary at 50 l. per Ann. is but 48 l. 2 s. 6 d. whose Factor by Table is 131849 for a common Year, which multiplied by 51 = 6.724299 = 6 l. 14 s. 5 $\frac{1}{4}$ d. the neat Salary for 51 Days.

But this Deduction for Tax and Charity may be made by multiplying of the gross Salary by .9625, a Decimal, and the Product will be the neat Salary. For Example, let the gross Salary be 6 l. how much will the neat Money be, with the Allowance of 9 d. in the Pound?

OPERATION.

9625 =	Factor.
6 l. =	Gross Salary.
l. s. d.	
5.7750 =	5 : 15 : 6 = neat Salary.
N	A Table

A Table of the Areas of Circles in Ale

Diam. in Ince	.0	.1	.2	.3	.4
12	.4011	.4078	.4146	.4214	.4282
13	.4707	.4780	.4853	.4927	.5001
14	.5459	.5537	.5616	.5695	.5775
15	.6266	.6350	.6435	.6520	.6604
16	.7130	.7219	.7309	.7400	.7491
17	.8049	.8144	.8239	.8335	.8432
18	.9024	.9124	.9225	.9327	.9429
19	1.0056	1.0160	1.0267	1.0374	1.0482
20	1.1140	1.1252	1.1364	1.1477	1.1590
21	1.2281	1.2400	1.2527	1.2655	1.2784
22	1.3480	1.3602	1.3726	1.3850	1.3974
23	1.4733	1.4861	1.4990	1.5120	1.5250
24	1.6042	1.6176	1.6310	1.6445	1.6580
25	1.7406	1.7546	1.7686	1.7827	1.7968
26	1.8827	1.8972	1.9117	1.9264	1.9411
27	2.0303	2.0456	2.0605	2.0757	2.0909
28	2.1835	2.1991	2.2148	2.2305	2.2463
29	2.3422	2.3584	2.3746	2.3908	2.4073
30	2.5066	2.5232	2.5401	2.5570	2.5739
31	2.6764	2.6938	2.7111	2.7285	2.7460
32	2.8519	2.8698	2.8877	2.9057	2.9237
33	3.0330	3.0514	3.0698	3.0884	3.1069
34	3.2196	3.2385	3.2576	3.2766	3.2958
35	3.4117	3.4312	3.4508	3.4705	3.4902
36	3.6095	3.6295	3.6497	3.6699	3.6901
37	3.8128	3.8334	3.8541	3.8749	3.8957
38	4.0217	4.0428	4.0641	4.0854	4.1068
39	4.2361	4.2578	4.2796	4.3015	4.3234
40	4.4561	4.4785	4.5008	4.5233	4.5457
41	4.6818	4.7046	4.7275	4.7505	4.7735
42	4.9129	4.9365	4.9598	4.9834	5.0069
43	5.1496	5.1736	5.1977	5.2218	5.2459
44	5.3920	5.4165	5.4413	5.4657	5.4904
45	5.6398	5.6649	5.6897	5.7153	5.7405
46	5.8933	5.9189	5.9446	5.9704	5.9962
47	6.1523	6.1785	6.2048	6.2311	6.2575
48	6.4169	6.4436	6.4705	6.4973	6.5243
49	6.6870	6.7143	6.7417	6.7692	6.7966
50	6.9628	6.9906	7.0186	7.0466	7.0746
51	7.2440	7.2725	7.3010	7.3295	7.3581
52	7.5309	7.5599	7.5890	7.6181	7.6472
53	7.8233	7.8529	7.8825	7.9122	7.9419
54	8.1214	8.1513	8.1816	8.2118	8.2421
55	8.4249	8.4556	8.4863	8.5171	8.5479
56	8.7341	8.7653	8.7966	8.8279	8.8593
57	9.0482	9.0806	9.1124	9.1443	9.1762
58	9.3691	9.4014	9.4338	9.4662	9.4983
59	9.6960	9.7287	9.7616	9.7945	9.8268

Gallons

Gallons from 12 to 59 Inches Diameter.

Diam. in Inchs.	.5	.6	.7	.8	.9
12	.4352	.4422	.4492	.4562	.4632
13	.5076	.5151	.5227	.5304	.5381
14	.5856	.5937	.6018	.6100	.6183
15	.6692	.6777	.6865	.6953	.7041
16	.7582	.7674	.7767	.7862	.7954
17	.8529	.8627	.8725	.8824	.8924
18	.9532	.9635	.9739	.9843	.9948
19	1.0590	1.0699	1.0808	1.0918	1.1029
20	1.1704	1.1819	1.1934	1.2049	1.2165
21	1.2874	1.2994	1.3114	1.3236	1.3357
22	1.4099	1.4225	1.4351	1.4478	1.4605
23	1.5380	1.5511	1.5643	1.5775	1.5908
24	1.6717	1.6854	1.6991	1.7129	1.7267
25	1.8110	1.8252	1.8395	1.8538	1.8681
26	1.9552	1.9706	1.9854	2.0003	2.0153
27	2.1062	2.1215	2.1369	2.1524	2.1679
28	2.2612	2.2780	2.2948	2.3116	2.3285
29	2.4237	2.4407	2.4577	2.4747	2.4917
30	2.5928	2.6078	2.6249	2.6410	2.6582
31	2.7635	2.7812	2.7987	2.8163	2.8341
32	2.9418	2.9599	2.9781	2.9963	3.0148
33	3.1256	3.1443	3.1630	3.1818	3.2007
34	3.3150	3.3342	3.3535	3.3728	3.3923
35	3.5099	3.5297	3.5496	3.5695	3.5894
36	3.7104	3.7308	3.7512	3.7717	3.7922
37	3.9165	3.9374	3.9584	3.9794	4.0005
38	4.1282	4.1496	4.1711	4.1927	4.2144
39	4.3454	4.3674	4.3895	4.4117	4.4338
40	4.5683	4.5908	4.6135	4.6362	4.6589
41	4.7966	4.8198	4.8430	4.8662	4.8895
42	5.0306	5.0543	5.0780	5.1019	5.1257
43	5.2702	5.2944	5.3187	5.3430	5.3675
44	5.5152	5.5400	5.5649	5.5898	5.6148
45	5.7659	5.7912	5.8167	5.8421	5.8677
46	6.0222	6.0480	6.0740	6.1000	6.1261
47	6.2839	6.3104	6.3369	6.3635	6.3901
48	6.5512	6.5782	6.6054	6.6325	6.6597
49	6.8242	6.8518	6.8794	6.9072	6.9349
50	7.1027	7.1309	7.1591	7.1873	7.2157
51	7.3868	7.4155	7.4443	7.4731	7.5020
52	7.6764	7.7057	7.7350	7.7644	7.7939
53	7.9717	8.0015	8.0314	8.0612	8.0912
54	8.2724	8.3028	8.3333	8.3638	8.3943
55	8.5788	8.6097	8.6407	8.6712	8.7019
56	8.8907	8.9222	8.9538	8.9854	9.0171
57	9.3082	9.3403	9.3724	9.4045	9.4368
58	9.5313	9.5639	9.5966	9.6293	9.6621
59	9.8600	9.8931	9.9263	9.9596	9.9930

A Table of the Areas of Circles in Ale

Diam. in Ines	0	,1	,2	,3	,4
60	10.0264	10.0598	10.0933	10.1269	10.1605
61	10.3634	10.3974	10.4314	10.4655	10.4997
62	10.7059	10.7405	10.7751	10.8098	10.8445
63	11.0541	11.0892	11.1244	11.1596	11.1949
64	11.4078	11.4434	11.4792	11.5150	11.5508
65	11.7670	11.8033	11.8396	11.8759	11.9123
66	12.1319	12.1687	12.2055	12.2424	12.2794
67	12.5023	12.5397	12.5771	12.6145	12.6520
68	12.8783	12.9162	12.9542	12.9922	13.0303
69	13.2599	13.2982	13.3368	13.3754	13.4140
70	13.6470	13.6860	13.7251	13.7642	13.8034
71	14.0397	14.0793	14.1189	14.1586	14.1983
72	14.4380	14.4781	14.5183	14.5585	14.5988
73	14.8418	14.8825	14.9232	14.9640	15.0049
74	15.2512	15.2925	15.3338	15.3754	15.4165
75	15.6662	15.7080	15.7499	15.7918	15.8337
76	16.0867	16.1291	16.1715	16.2140	16.2565
77	16.5129	16.5558	16.5988	16.6418	16.6849
78	16.9445	16.9880	17.0316	17.0751	17.1188
79	17.3818	17.4258	17.4699	17.5141	17.5583
80	17.8246	17.8692	17.9139	17.9586	18.0033
81	18.2731	18.3182	18.3634	18.4086	18.4540
82	18.7270	18.7727	18.8185	18.8643	18.9102
83	19.1866	19.2328	19.2791	19.3255	19.3719
84	19.6517	19.6985	19.7454	19.7923	19.8393
85	20.1223	20.1697	20.2171	20.2646	20.3122
86	20.5986	20.6465	20.6945	20.7426	20.7907
87	21.0806	21.1289	21.1775	21.2261	21.2747
88	21.5678	21.6169	21.6660	21.7151	21.7643
89	22.0602	22.1104	22.1600	22.2098	22.2595
90	22.5599	22.6095	22.6597	22.7100	22.7603
91	23.0634	23.1141	23.1649	23.2157	23.2667
92	23.5731	23.6244	23.6757	23.7271	23.7785
93	24.0889	24.1408	24.1920	24.2440	24.2960
94	24.6091	24.6615	24.7140	24.7665	24.8190
95	25.1355	25.1885	25.2415	25.2945	25.3376
96	25.6675	25.7210	25.7745	25.8282	25.8818
97	26.2050	26.2591	26.3132	26.3673	26.4216
98	26.7481	26.8037	26.8574	26.9121	26.9669
99	27.2968	27.3519	27.4072	27.4625	27.5178
100	27.8510	27.9067	27.9625	28.0184	28.0743
101	28.4108	28.4671	28.5236	28.5798	28.6363
102	28.9761	29.0330	29.0899	29.1469	29.2039

Gallons from 60 to 102 Inches Diameter.

Diam. in Ines.	.5	.6	.7	.8	.9
60	10.1948	10.2179	10.2617	10.2953	10.3294
61	10.3339	10.3682	10.6026	10.6370	10.6714
62	10.8793	10.9141	10.9490	10.9840	11.0190
63	11.2302	11.2656	11.3012	11.3366	11.3721
64	11.5867	11.6227	11.6587	11.6947	11.7307
65	11.9488	11.9853	12.0219	12.0585	12.0952
66	12.3164	12.3535	12.3906	12.4278	12.4650
67	12.6896	12.7272	12.7649	12.8027	12.8405
68	13.0684	13.1066	13.1448	13.1831	13.2215
69	13.4527	13.4915	13.5297	13.5681	13.6080
70	13.8426	13.8819	13.9208	13.9607	14.0002
71	14.2382	14.2780	14.3174	14.3579	14.3979
72	14.6398	14.6796	14.7191	14.7606	14.8012
73	15.0458	15.0866	15.1278	15.1689	15.2100
74	15.4580	15.4990	15.5412	15.5837	15.6244
75	15.8758	15.9178	15.9600	16.0022	16.0444
76	16.2991	16.3417	16.3844	16.4272	16.4700
77	16.7280	16.7712	16.8145	16.8578	16.9011
78	17.1625	17.2062	17.2500	17.2939	17.3378
79	17.6025	17.6468	17.6912	17.7356	17.7801
80	18.0481	18.0930	18.1379	18.1829	18.2280
81	18.4993	18.5448	18.5902	18.6358	18.6814
82	18.9561	19.0021	19.0482	19.0942	19.1403
83	19.4184	19.4650	19.5115	19.5581	19.6049
84	19.8863	19.9334	19.9806	20.0278	20.0750
85	20.3598	20.4074	20.4551	20.5028	20.5507
86	20.8388	20.8870	20.9353	20.9836	21.0320
87	21.3234	21.3722	21.4210	21.4699	21.5188
88	21.8136	21.8629	21.9123	21.9617	22.0112
89	22.3093	22.3592	22.4092	22.4592	22.5092
90	22.8107	22.8611	22.9116	22.9621	23.0128
91	23.3176	23.3685	23.4196	23.4707	23.5210
92	23.8300	23.8816	23.9332	23.9848	24.0367
93	24.3480	24.4002	24.4523	24.5045	24.5568
94	24.8716	24.9243	24.9770	25.0298	25.0826
95	25.4008	25.4540	25.5073	25.5606	25.6140
96	25.9355	25.9893	26.0432	26.0971	26.1510
97	26.4759	26.5302	26.5846	26.6390	26.6935
98	27.0127	27.0766	27.1316	27.1866	27.2416
99	27.5732	27.6286	27.6841	27.7397	27.7953
100	28.1302	28.1862	28.2423	28.2984	28.3546
101	28.6928	28.7494	28.8060	28.8627	28.9194
102	29.2610	29.3182	29.3753	29.4325	29.4898

A Table of the Areas of Circles in Ale

Diam. in Incht.	30	31	32	33	34
103	29.5472	29.6045	29.6620	29.7195	29.7771
104	30.1236	30.1816	30.2396	30.2977	30.3558
105	30.7057	30.7642	30.8228	30.8814	30.9401
106	31.2936	31.3525	31.4116	31.4708	31.5300
107	31.8866	31.9462	32.0059	32.0657	32.1255
108	32.4854	32.5456	32.6058	32.6661	32.7265
109	33.0898	33.1509	33.2113	33.2722	33.3331
110	33.6997	33.7610	33.8224	33.8838	33.9452
111	34.3152	34.3771	34.4390	34.5010	34.5630
112	34.9363	34.9987	35.0612	35.1237	35.1863
113	35.5629	35.6259	35.6889	35.7520	35.8152
114	36.1952	36.2587	36.3223	36.3859	36.4496
115	36.8339	36.8970	36.9612	37.0254	37.0896
116	37.4783	37.5409	37.6036	37.6670	37.7302
117	38.1252	38.1904	38.2557	38.3210	38.3864
118	38.7797	38.8459	38.9113	38.9772	39.0431
119	39.4398	39.5061	39.5725	39.6389	39.7054
120	40.1054	40.1723	40.2392	40.3061	40.3733
121	40.7766	40.8442	40.9116	40.9791	41.0467
122	41.4534	41.5214	41.5895	41.6575	41.7257
123	42.1358	42.2043	42.2729	42.3416	42.4103
124	42.8237	42.8928	42.9619	43.0312	43.1004
125	43.5172	43.5868	43.6560	43.7256	43.7961
126	44.2162	44.2865	44.3567	44.4271	44.4974
127	44.9209	44.9916	45.0623	45.1330	45.2043
128	45.6311	45.7024	45.7738	45.8452	45.9167
129	46.3468	46.4187	46.4907	46.5627	46.6347
130	47.0682	47.1406	47.2131	47.2857	47.3583
131	47.7951	47.8681	47.9412	48.0143	48.0874
132	48.5276	48.6011	48.6747	48.7484	48.8221
133	49.1656	49.2397	49.3139	49.388	49.4624
134	50.0093	50.0839	50.1586	50.2334	50.3083
135	50.7584	50.8342	50.9099	50.9842	51.0600
136	51.5132	51.5890	51.6650	51.7410	51.8170
137	52.2740	52.3500	52.4262	52.5030	52.5792
138	53.0394	53.1164	53.1933	53.2703	53.3474
139	53.8109	53.8884	53.9659	54.0434	54.1211
140	54.5880	54.6660	54.7440	54.8222	54.9003
141	55.3706	55.4491	55.5268	55.6064	55.6852
142	56.1587	56.2379	56.3170	56.3963	56.4756
143	56.9525	57.0322	57.1119	57.1917	57.2716
144	57.7518	57.8322	57.9124	57.9927	58.0731
145	58.5567	58.6375	58.7184	58.7993	58.8802
146	59.3672	59.4485	59.5299	59.6114	59.6929

Gallons from 102 to 146 Inches Diameter.

Diam. in In.	.5	.6	.7	.8	.9
103	29.8347	29.8924	29.9501	30.0079	30.0657
104	30.4140	30.4722	30.5305	30.5889	30.6473
105	30.9939	31.0577	31.1165	31.1754	31.2344
106	31.5893	31.6487	31.7081	31.7675	31.8270
107	32.1853	32.2452	32.3052	32.3652	32.4252
108	32.7869	32.8474	32.9079	32.9685	33.0291
109	33.3940	33.4551	33.5161	33.5773	33.6385
110	34.0068	34.0683	34.1300	34.1917	34.2534
111	34.6251	34.6872	34.7494	34.8116	34.8739
112	35.2489	35.3116	35.3744	35.4372	35.5000
113	35.8784	35.9416	36.0049	36.0683	36.1317
114	36.5134	36.5772	36.6418	36.7049	36.7689
115	37.1539	37.2183	37.2827	37.3472	37.4117
116	37.8001	37.8650	37.9300	37.9950	38.0601
117	38.4518	38.5173	38.5828	38.6484	38.7142
118	39.1091	39.1751	39.2412	39.3073	39.3735
119	39.7719	39.8385	39.9052	39.9719	40.0386
120	40.4403	40.5075	40.5747	40.6420	40.7093
121	41.1143	41.1820	41.2498	41.3176	41.3855
122	41.7939	41.8622	41.9305	41.9989	42.0673
123	42.4790	42.5479	42.6167	42.6858	42.7547
124	43.1697	43.2391	43.3086	43.3780	43.4476
125	43.8660	43.9360	44.0059	44.0760	44.1461
126	44.5679	44.6384	44.7089	44.7795	44.8502
127	45.2753	45.3463	45.4174	45.4886	45.5598
128	45.9883	46.0599	46.1315	46.2032	46.2750
129	46.7068	46.7790	46.8512	46.9235	46.9958
130	47.4309	47.5037	47.5764	47.6493	47.7222
131	48.1606	48.2339	48.3073	48.3806	48.4541
132	48.8959	48.9697	49.0436	49.1176	49.1916
133	49.6367	49.7111	49.7856	49.8601	49.9346
134	50.3832	50.4581	50.5331	50.6082	50.6833
135	51.1351	51.2110	51.2861	51.3620	51.4374
136	51.8930	51.9690	52.0450	52.1210	52.1972
137	52.6560	52.7324	52.8090	52.8860	52.9630
138	53.4245	53.5017	53.5789	53.6562	53.7335
139	54.1987	54.2765	54.3543	54.4322	54.5100
140	54.9786	55.0569	55.1325	55.2136	55.2921
141	55.7642	55.8428	55.9217	56.0007	56.0797
142	56.5549	56.6343	56.7138	56.7933	56.8729
143	57.3515	57.4314	57.5114	57.5915	57.6716
144	58.1536	58.2341	58.3147	58.3953	58.4760
145	58.9613	59.0423	59.1235	59.2047	59.2859
146	59.7745	59.8561	59.9378	60.0196	60.1014

184 *Explanation of the Table of Ale Areas.*

Of the Table of Ale Areas.

In the first Column under the Words *Diam.* in Inches seek for the even Inches of any given Diameter from 12 to 146 Inches, and right against it under 0 in the second Column you will find the Area to the ten thousandth Part of a Gallon in Ale Measure. But if the given Diameter consists of Inches and Tenths of an Inch, then seek the even Inches of the Diameter in either the 1st or 7th Column, and the Tenths in any of the other 9 Columns at the Head of the Table, and at the Angle of meeting you will find the Area as before.

EXAMPLE I.

Suppose it were required to find the Ale Area of a Circle whose Diameter was 60 Inches. I look for 60 in the 1st Column, and right against it in the 2d Column under 0 I find $10.0264 =$ the Area in Ale Gallons.

EXAMPLE II.

Suppose the given Diameter were 75.3 Inches, what is the Area in Ale Gallons?

I find 75 in the 1st Column, and the 3 Tenths at the Head of the Table, then at the Angle of meeting I find $15.7918 =$ the Area in Ale Gallons required.

EXAMPLE III.

Suppose the given Diameter were 78.7 Inches: To find the Area in Ale Gallons.

I find 78 in the 7th Column, and the 7 Tenths at the Head of the Table, then at the Angle of meeting is 17.25 , the Area in Ale Gallons required.

And thus may the Area of any other Diameter within the Compass of the Table be found. But if you should want to know the Ale Area of a Circle of a less or greater Diameter than any contained in the Table, you may find it by the following Directions:

1. When the given Diameter is under 12 Inches, seek for the Area of double that Diameter and divide it by 4, and the Quotient will $=$ the Area required.

2. If the given Diameter exceeds 146, seek for the Area of half that Diameter in the Table and multiply

Explanation of the Tables of Ale Areas 185

multiply it by 4, and that Product will be equal to the Area sought. For Example in the 1st Case, suppose it were required to find the Ale Area of a Circle whose Diameter is 6 Inches, I look in the Table for the Area of 12 Inches, the Double of 6, which I find to be = .4011, which divided by 4 = .100275 = Area required.

Example in the second Case.

Suppose the given Diameter 160 Inches, what is the Area thereof in Ale Gallons?

I seek for 80 = Half the given Diameter in the Table, where I find the Area = 17.8246, which multiplied by 4 is = 71.2984 = to the Area required.

And in this Manner the Area of any other Diameter that exceeds the Bounds of the Table may be found.

A Table to convert Gallons into Barrels, & contra.

Bar.	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	Bar.	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
0	0	8.5	17	25.5	11	714	722.5	731	739.5
1	34	42.5	51	59.5	12	748	756.5	765	773.5
2	68	76.5	85	93.5	13	782	790.5	799	807.5
3	102	110.5	119	127.5	14	816	824.5	833	841.5
4	136	144.5	153	161.5	15	850	858.5	867	875.5
5	170	178.5	187	195.5	16	884	892.5	901	909.5
6	204	212.5	221	229.5	17	918	926.5	935	943.5
7	238	246.5	255	263.5	18	952	960.5	969	977.5
8	272	280.5	289	297.5	19	986	994.5	1003	1011.5
9	306	314.5	323	331.5	20	1020	1028.5	1037	1045.5
10	340	348.5	357	365.5	21	1054	1062.5	1071	1079.5
11	374	382.5	391	399.5	22	1088	1096.5	1105	1113.5
12	408	416.5	425	433.5	23	1122	1130.5	1139	1147.5
13	442	450.5	459	467.5	24	1156	1164.5	1173	1181.5
14	476	484.5	493	501.5	25	1190	1198.5	1207	1215.5
15	510	518.5	527	535.5	26	1224	1232.5	1241	1249.5
16	544	552.5	561	569.5	27	1258	1266.5	1275	1283.5
17	578	586.5	595	603.5	28	1292	1300.5	1309	1317.5
18	612	620.5	629	637.5	29	1326	1334.5	1343	1351.5
19	646	654.5	663	671.5	30	1360	1368.5	1377	1385.5
20	680	688.5	697	705.5	31	1394	1402.5	1411	1419.5

To

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To convert any Number of Gallons into Barrels seek the given Number of Gallons, or the nearest less, in any of the Columns except that under Barrels, and right against it to the Left Hand under Barrels are the Barrels; and at the Head of the Table in the same Column the Quarters, if any.

EXAMPLE I.

Suppose it were required to reduce 255 Gallons to Barrels, &c.

I find 255 in the fourth Column under $\frac{1}{2}$, and right against it under Barrels is 7. which added together is 7 B. and $\frac{1}{2} = 255$ Gallons.

EXAMPLE II.

Suppose it were required to reduce 250 Gallons into Barrels, &c.

I find in the third Column the nearest less to 250 is 246.5, which is 3.5 less than the given Number. Against it on the Left Hand under Barrels, is 7, and above it at the Head is $\frac{3}{4}$: So the Whole is 7 B. 1 F. and 3.5 G. = 250 Gallons; which is so easy that it would be needless to add any more Examples.

•• The following Paragraphs were intended to be inserted at the latter End of Art. i. Chap. iv. but it being mislaid when that Sheet was printed off, is the Reason of its being placed here.

There are now made a more convenient Sort of Sliding Rules than that before described, which were first invented by Mr. Verie, then Collector of Excise. This Rule consists of four Slides with a single Radius, and is so contrived that two of the Slides work together; by which Means one of this Sort made 6 Inches long answers another made the common Way of 12 Inches, or one of 9 Inches to another of 18 Inches; whereby the Divisions are enlarged, and the Rule rendered much more portable.

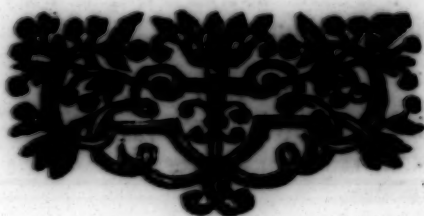
On each of the 4 Slides is put the Line of Numbers, commonly called Gunter's Line, with all the usual Marks and Dots as on the common Rules; two of which are for sliding together between the Line D which is broken into two Parts, and the Lines A and M D; the other two are for sliding between the Lines of Segments, which are also broken into two Parts. And although the Line D and the two Lines of Segments are broken into two Parts, yet

To convert Gallons of Ale into Barrels. 187

yet they are so fitly placed on the Sides of the Rule that the two Slides may slide together between them in such a Manner as that any Question may be as readily solved as if they were in one continued Line.

Underneath the Slides are put the Lines of Varieties, the Divisors and Gagepoints; and also the several Factors for reducing of Goods of one Denomination to its equivalent Value in those of another.

There is also a Table of the Malt Divisor 2150, which is doubled, trebled, &c. to 9 Times, for estimating the Content of a Floor of Malt. If the Rule before described be thoroughly understood, this will be known at the first Sight.



APPENDIX:

APPENDIX.

ART. I.

To gage a Still, and find the Content thereof on every Inch of its Depth.

RULE.

1. **S**UPPOSE the Still to be divided into three Parts by the Lines $c d c$ and $A o A$; then that Part from the Collar $c c$ to the Place where the Nails join it to the Body at $c c$ is supposed to be globular, and may be gaged as the middle Frustum or Zone of a Sphere, but in Practice by taking Mean Diameters in the Middle of every 3 or 4 Inches of its Depth.

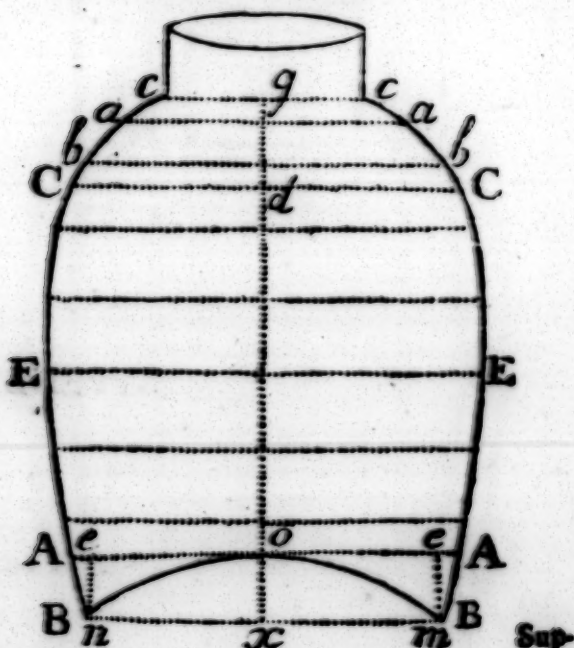
2. The middle Part or Body of the Still, *viz.* from $c c$ to $A A$, is taken for the middle Frustum of a Spheroid, and as such may be gaged, but in Practice by taking of Mean Diameters in the Middle of every 6 Inches of its Depth.

3. The lower Part $A o A B o B$, or that Part which includes the Crown, may be gaged as the like Part of the Copper; or in Practice by pouring in of Water from a Gallon, or some known Measure, till the Crown is covered, which will be when the Water touches the Point o .

EXAMPLE.

Admit the following Still was to be gaged, and the Content required at every Inch of the Depth in Wine Gallons.

A STILL.



Suppose with an Instrument I found the Depth of the Body of the Still $d e = 30$ Inches, and that of the globular Part $g d = 7$ Inches, then the whole Depth from the Top of the Crown to the Collar will be 37 Inches $= g o$; and suppose with a Sliding Cane I found the first Mean Diameter $a s$ in the Middle of the upper 3 Inches of the globular Part $= 54$ Inches, and the second Mean Diameter $b b$ in the Middle of the next 4 Inches $= 60.8$, as is set down in the second Column of the following Table.

Next I take Mean Diameters in the Middle of every 6 Inches of the spheroidal Part $A E C C E A$, and set them also down as in the second Column of the Table.

And suppose I found by Measure that the Quantity of Water to cover the Crown is $= 34$ Gallons, which I set down in the fourth Column of the Table.

By the Tables of Wine Areas in Pages 194, &c. I find the Area of each Diameter and place them against their respective Diameters in the third Column of the Table.

Then I multiply the several Areas by the respective Depths to which they belong, and set the Products in the fourth Column, which are the Contents of the separate Parts; whose Sum, with the Quantity of Liquor to cover the Crown, will be equal to $606.842 =$ the whole Content of the Still.

Depth.	Diam.	Area.	Cont.
3	54.0	9.914	29.742
4	60.8	12.568	50.272
6	67.0	15.263	91.578
6	71.0	17.140	102.840
6	72.0	17.626	105.756
6	70.2	16.755	100.530
6	67.2	15.354	92.124
	To cov. Cr		34.
37	Whole Cont.		606.842

To find how much the Still will hold on every Inch proceed as follows:

Whole Cont.	Continued.	Continued.
606.842	435.250	209.899
Sub. 9.914	Note 17.140	16.755
1 = 596.928	14 = 418.110	27 = 193.144
9.914	17.140	16.755
2 = 587.014	15 = 400.970	28 = 176.389
9.914	17.140	16.755
3 = 577.100	16 = 383.830	29 = 159.634
Note 12.568	17.140	16.755
4 = 564.532	17 = 366.690	30 = 142.879
12.568	17.140	16.755
5 = 551.964	18 = 349.550	31 = 126.124
12.568	17.140	Note 15.354
6 = 539.396	19 = 332.410	32 = 110.770
12.568	Note 17.626	15.354
7 = 526.828	20 = 314.784	33 = 95.416
Note 15.263	17.626	15.354
8 = 511.565	21 = 297.158	34 = 80.062
15.263	17.626	15.354
9 = 496.302	22 = 279.532	35 = 64.708
15.263	17.626	15.354
10 = 481.039	23 = 261.906	36 = 49.354
15.263	17.626	15.354
11 = 465.777	24 = 244.280	37 = 34.000
15.263	17.626	Crp. 34
12 = 450.513	25 = 226.654	00000
15.263	Note 16.755	1
13 = 435.250	26 = 209.899	

A Table

A Table shewing the Content of the Still at every dry Inch to the nearest even Gallon, collected from the preceding Work.

Dry In ^{ch}	Galls.	Dry In ^{ch}	Galls.	Dry In ^{ch}	Galls.
Full	607	13	435	26	210
1	597	14	418	27	193
2	587	15	401	28	176
3	577	16	384	29	160
4	565	17	367	30	143
5	553	18	350	31	126
6	539	19	332	32	111
7	527	20	315	33	95
8	512	21	297	34	80
9	496	22	280	35	65
10	481	23	262	36	49
11	466	24	244	37	34
12	451	25	227	Cov. C ^h	34

ART. II.

To find the Content of the Still without taking of Mean Diameters.

1. **F**OR the Content of the globular Part *c c c c*, supposing it to be the middle Frustrum or Zone of a Sphere.

RULE.

To half the Sum of the Squares of the Top and Bottom Diameters *c c* and *c c* add two Thirds of the Square of the Altitude *g d*, and that Sum multiply by the Altitude; this Product divide by 294.12, and the Quotient will give the Content of that Part in Wine Gallons. *Vide Simpson's Fluxions, Page 212.*

EXAMPLE.

Suppose the

Top Diam. <i>c c</i> = 50.5	} Inca. {	Sq. {	2550.25
Bot. Diam. <i>c c</i> = 64		is {	4096.
Sum of the Squares	—	—	= 6646.25
Half Sum	—	—	= 3323.125
Two Thirds of Square of Alt. 7	—	—	= 32.666
Sum	—	—	= 3355.791
Multiply by the Altitude	—	—	= 7
Product	—	—	= 23490.537

For the Content.

294.12)23490.537(79.867 Wine Gallons.

2. For

2. For the Content of the spheroidal Part AEC CBA , or the Body of the Still.

RULE.

This is found by Art. iii. of Chap. xi. as the first Variety of Casks.

EXAMPLE.

Let the

$$\begin{array}{lcl} \text{Greatest Diameter } EE \text{ in the Middle} & \} & = 72 \\ \text{of the Body} & \text{---} & \\ \text{Diam. } AA = CC \text{ at the End} & \text{---} & = 64 \\ \text{Length } ds & \text{---} & = 30 \end{array} \left. \vphantom{\begin{array}{l} 72 \\ 64 \\ 30 \end{array}} \right\} \text{Inches.}$$

OPERATION.

$$\text{To twice the Square of } EE \ 72 = 10368$$

$$\text{Add the Square of } CC \ 64 = 4096$$

$$\text{Sum} \text{ ---} = 14464$$

$$\text{Multiply by the Altitude } ds = 30 \text{ Cent.}$$

$$\text{Divide by 3 Times } \left. \vphantom{\begin{array}{l} 14464 \\ 30 \end{array}} \right\} = 882.36 \text{ } 433920(491.772 \text{ the Circ. Divif.}$$

3. To find the Content of the Part $AQABQB$, or the Quantity of Liquor to cover the Crown.

RULE.

This is found by the Directions in Art. ii. Chap. x. for the like Part of the Copper.

EXAMPLE.

Let the

$$\begin{array}{lcl} \text{Diam. } AA \text{ at Top of Crown} = 64 & \} & \text{Area} \left\{ \begin{array}{l} 13.926 \\ .085 \end{array} \right. \\ \text{Alt. of Crown } ex = en = em = 5 & \} & \\ \text{Add } \frac{1}{7} \text{ of the Area of } 5 & \text{---} & = .025 \end{array}$$

$$\text{Sum subtract from the Area of the } \left. \vphantom{\begin{array}{l} 13.926 \\ .085 \end{array}} \right\} = .110$$

$$\text{Remainder} \text{ ---} = 13.816$$

$$\text{Multiply by } \frac{1}{2} \text{ the Height of Crown} = 2.5$$

$$69080$$

$$27632$$

$$\text{Quantity of Liquor to cover the Crn.} = 34.5400$$

$$\text{The Body of the Still is} \text{ ---} = 491.772$$

$$\text{Globular Part is} \text{ ---} = 79.867$$

$$\text{The whole Content is} \text{ ---} = 606.1790$$

The Content found in this Manner is within a Gallon of that found before by taking of Mean Diameters.

A Cask

A Case Table for Sweets at 12s. per Barrel.

Galls.	s.	d.	7 ^{ths} of a Far.	Galls.	s.	d.	7 ^{ths} of a Far.	Barrels.	L.	s.
1	0	1	4	16	6	1	4	2	1	4
2	0	2	1	17	6	5	6	3	1	16
3	0	3	5	18	6	10	1	4	2	8
4	0	4	2	19	7	2	3	5	3	0
5	0	9	4	20	7	7	5	6	3	12
6	1	1	6	21	8	0	0	7	4	4
7	1	6	1	22	8	4	2	8	4	16
8	1	10	3	23	8	9	4	9	5	8
9	2	3	5	24	9	1	6	10	6	0
10	2	8	0	25	9	6	1	20	12	0
11	3	0	2	26	9	10	3	30	18	0
12	3	5	4	27	10	3	5	40	24	0
13	3	9	6	28	10	8	0	50	30	0
14	4	2	1	29	11	0	2	60	36	0
15	4	6	3	30	11	5	4	70	42	0
16	4	11	5	31	11	9	6	80	48	0
17	5	4	0	32	12	0	0	90	54	0
18	5	8	2	33	12	0	0	100	60	0

Of the Use of the above Table.

1. If it be required to find the Duty of any Number of Gallons and Quarters of Sweets,

Find the given Number of Gallons in the first or fifth Column under the Word Gallons, then against it on the Right Hand is the Duty in Shillings, Pence, and 7th Parts of a Farthing: If there be any Quarts find them in the first Column, and right against them is the Duty, which added to the Duty of the Gallons found before will give the Duty of the Number required.

2. If the given Number consists of Barrels, Gallons, and Quarts, then seek the Barrels in the 9th Column under the Word Barrels, and against them on the Right Hand is the Duty, which added to the Duty of the Gallons and Quarts found by the first Case, will give the Duty of the Number required. An Example or two will make this quite plain.

EXAMPLE I.

What is the Duty of $7\frac{1}{2}$ Gallons of Sweets?

s. d. pt.

Against 7 in the first Column is 2: 8 :0

Against $\frac{1}{2}$ in the same Column is 0: $2\frac{1}{2}$:1

Their Sum = Duty required = 2:10: $\frac{1}{2}$

EXAMPLE II.

What is the Duty of 10 B. $5\frac{1}{2}$ Gs. of Sweets?

l. s. d. pt.

Against 10 in the 9th Col. under Barr. is 6:0: 0 :0

Against 5 in the 1st Col. under Gall. is 0:0:10: $\frac{1}{2}$:3

Against $\frac{1}{2}$ in the same Column is — 0:0: $2\frac{1}{2}$:1

Sum is equal to the Duty required = 6:2: 1 :4

0

4. 4

4. A Table of the Areas of Circles in Wane.

Diam. in Ines.	.0	.1	.2	.3	.4
1	0.0034	0.0041	0.0049	0.0057	0.0067
2	0.0136	0.0150	0.0165	0.0180	0.0196
3	0.0306	0.0327	0.0348	0.0370	0.0393
4	0.0544	0.0572	0.0600	0.0629	0.0658
5	0.0850	0.0884	0.0919	0.0955	0.0991
6	0.1214	0.1265	0.1307	0.1349	0.1393
7	0.1666	0.1714	0.1763	0.1812	0.1862
8	0.2176	0.2231	0.2286	0.2342	0.2399
9	0.2754	0.2816	0.2878	0.2941	0.3004
10	0.3400	0.3468	0.3537	0.3607	0.3677
11	0.4114	0.4189	0.4265	0.4341	0.4419
12	0.4896	0.4978	0.5061	0.5144	0.5228
13	0.5746	0.5835	0.5924	0.6014	0.6105
14	0.6664	0.6760	0.6856	0.6953	0.7050
15	0.7650	0.7752	0.7855	0.7959	0.8063
16	0.8704	0.8813	0.8923	0.9033	0.9145
17	0.9826	0.9932	1.0059	1.0176	1.0294
18	1.1016	1.1139	1.1262	1.1386	1.1511
19	1.2274	1.2404	1.2534	1.2665	1.2796
20	1.3600	1.3739	1.3879	1.4011	1.4149
21	1.4994	1.5137	1.5281	1.5425	1.5571
22	1.6456	1.6606	1.6757	1.6908	1.7060
23	1.7986	1.8143	1.8300	1.8451	1.8617
24	1.9584	1.9748	1.9912	2.0077	2.0242
25	2.1250	2.1420	2.1591	2.1763	2.1935
26	2.2984	2.3161	2.3339	2.3517	2.3697
27	2.4786	2.4970	2.5155	2.5340	2.5526
28	2.6656	2.6847	2.7038	2.7230	2.7422
29	2.8594	2.8792	2.8990	2.9189	2.9388
30	3.0600	3.0804	3.1009	3.1215	3.1421
31	3.2674	3.2885	3.3097	3.3309	3.3523
32	3.4816	3.5034	3.5253	3.5472	3.5692
33	3.7026	3.7251	3.7476	3.7702	3.7929
34	3.9304	3.9536	3.9768	4.0001	4.0234
35	4.1650	4.1888	4.2127	4.2367	4.2607
36	4.4064	4.4309	4.4555	4.4801	4.5049
37	4.6546	4.6798	4.7051	4.7304	4.7558
38	4.9096	4.9355	4.9614	4.9874	5.0135
39	5.1714	5.1980	5.2246	5.2513	5.2780
40	5.4400	5.4672	5.4945	5.5219	5.5493
41	5.7154	5.7433	5.7713	5.7993	5.8275
42	5.9976	6.0262	6.0549	6.0836	6.1124
43	6.2866	6.3159	6.3452	6.3746	6.4041
44	6.5824	6.6124	6.6424	6.6725	6.7026
45	6.8850	6.9156	6.9463	6.9771	7.0079

Gallons from 1 to 46 Inches Diameter.

Diam. in Incht.	5	6	7	8	9
1	0.0077	0.0087	0.0098	0.0110	0.0123
2	0.0213	0.0230	0.0248	0.0267	0.0286
3	0.0417	0.0441	0.0465	0.0491	0.0517
4	0.0689	0.0719	0.0751	0.0781	0.0816
5	0.1029	0.1066	0.1105	0.1144	0.1184
6	0.1437	0.1481	0.1526	0.1572	0.1619
7	0.1913	0.1964	0.2016	0.2069	0.2123
8	0.2457	0.2515	0.2573	0.2633	0.2693
9	0.3069	0.3133	0.3199	0.3265	0.3332
10	0.3749	0.3820	0.3893	0.3966	0.4040
11	0.4497	0.4575	0.4654	0.4734	0.4815
12	0.5313	0.5398	0.5484	0.5571	0.5658
13	0.6197	0.6289	0.6381	0.6475	0.6569
14	0.7149	0.7247	0.7347	0.7447	0.7548
15	0.8169	0.8274	0.8381	0.8488	0.8596
16	0.9257	0.9369	0.9482	0.9596	0.9711
17	1.0413	1.0532	1.0652	1.0773	1.0894
18	1.1637	1.1763	1.1889	1.2017	1.2145
19	1.2929	1.3061	1.3195	1.3329	1.3464
20	1.4289	1.4428	1.4569	1.4710	1.4852
21	1.5717	1.5863	1.6010	1.6158	1.6307
22	1.7213	1.7366	1.7520	1.7675	1.7830
23	1.8777	1.8937	1.9097	1.9259	1.9421
24	2.0409	2.0575	2.0743	2.0911	2.1080
25	2.2109	2.2282	2.2457	2.2632	2.2808
26	2.3877	2.4057	2.4238	2.4420	2.4603
27	2.5713	2.5900	2.6088	2.6277	2.6466
28	2.7617	2.7811	2.8005	2.8101	2.8396
29	2.9589	2.9789	2.9991	3.0193	3.0396
30	3.1629	3.1836	3.2045	3.2254	3.2464
31	3.3737	3.3951	3.4166	3.4382	3.4599
32	3.5913	3.6134	3.6356	3.6579	3.6802
33	3.8157	3.8385	3.8613	3.8843	3.9073
34	4.0469	4.0703	4.0939	4.1175	4.1412
35	4.2849	4.3090	4.3333	4.3576	4.3820
36	4.5297	4.5545	4.5794	4.6044	4.6295
37	4.7813	4.8068	4.8324	4.8585	4.8838
38	5.0397	5.0659	5.0921	5.1185	5.1449
39	5.3049	5.3317	5.3587	5.3857	5.4128
40	5.5769	5.6044	5.6321	5.6598	5.6876
41	5.8557	5.8839	5.9122	5.9406	5.9691
42	6.1413	6.1702	6.1992	6.2283	6.2574
43	6.4337	6.4633	6.4929	6.5227	6.5525
44	6.7329	6.7631	6.7935	6.8239	6.8544
45	7.0389	7.0698	7.1009	7.1320	7.1632

A Table of the Areas of Circles in Wine

Diam. in Inchs.	.0	.1	.2	.3	.4
46	7.1944	7.2257	7.2571	7.2885	7.3201
47	7.5106	7.5426	7.5747	7.6068	7.6390
48	7.8336	7.8663	7.8990	7.9318	7.9647
49	8.1534	8.1968	8.2302	8.2637	8.2972
50	8.5000	8.5340	8.5681	8.6023	8.6365
51	8.8434	8.8781	8.9129	8.9477	8.9827
52	9.1935	9.2290	9.2645	9.3000	9.3356
53	9.5506	9.5867	9.6228	9.6590	9.6953
54	9.9144	9.9512	9.9880	10.0245	10.0618
55	10.2850	10.3224	10.3599	10.3975	10.4351
56	10.6624	10.7005	10.7387	10.7769	10.8153
57	11.0466	11.0854	11.1243	11.1632	11.2022
58	11.4376	11.4771	11.5166	11.5562	11.5959
59	11.8354	11.8756	11.9158	11.9561	11.9964
60	12.2400	12.2808	12.3217	12.3627	12.4037
61	12.6516	12.6929	12.7344	12.7754	12.8180
62	13.0696	13.1079	13.1460	13.1845	13.2238
63	13.4948	13.5335	13.5724	13.6114	13.6504
64	13.9264	13.9658	14.0053	14.0451	14.0848
65	14.3652	14.4050	14.4453	14.4856	14.5244
66	14.8104	14.8509	14.8904	14.9309	14.9704
67	15.2628	15.3037	15.3440	15.3851	15.4252
68	15.7216	15.7632	15.8036	15.8450	15.8852
69	16.1876	16.2296	16.2712	16.3137	16.3556
70	16.6551	16.6977	16.7352	16.7751	16.8150
71	17.1399	17.1827	17.2236	17.2655	17.3052
72	17.6256	17.6694	17.7125	17.7556	17.7972
73	18.1128	18.1569	18.2000	18.2431	18.2852
74	18.6184	18.6632	18.7072	18.7514	18.7952
75	19.1252	19.1704	19.2152	19.2596	19.3038
76	19.6384	19.6843	19.7300	19.7756	19.8204
77	20.1588	20.2050	20.2508	20.2964	20.3412
78	20.6856	20.7326	20.7792	20.8256	20.8712
79	21.2196	21.2669	21.3136	21.3596	21.4052
80	21.7600	21.8080	21.8552	21.9012	21.9472
81	22.3076	22.3559	22.4036	22.4512	22.4984
82	22.8616	22.9107	22.9588	23.0064	23.0536
83	23.4228	23.4722	23.5208	23.5688	23.6164
84	23.9904	24.0405	24.0900	24.1388	24.1864
85	24.5652	24.6160	24.6652	24.7136	24.7612
86	25.1464	25.1975	25.2480	25.2972	25.3456
87	25.7348	25.7862	25.8368	25.8864	25.9348
88	26.3296	26.3817	26.4328	26.4824	26.5308
89	26.9316	26.9840	27.0352	27.0848	27.1328
90	27.5400	27.5931	27.6452	27.6952	27.7432

APPENDIX. 197

Gallons from 46 to 90 Inches Diameter.

Diam. in Ines.	45	46	47	48	49
46	7.1517	7.3833	7.4150	7.4468	7.4787
47	7.6713	7.7036	7.7300	7.7685	7.8010
48	7.9977	8.0307	8.0637	8.0969	8.1301
49	8.3309	8.3641	8.3983	8.4321	8.4660
50	8.6709	8.7052	8.7391	8.7741	8.8082
51	9.0177	9.0527	9.0878	9.1230	9.1583
52	9.3713	9.4070	9.4428	9.4787	9.5146
53	9.7317	9.7681	9.8045	9.8411	9.8777
54	10.0989	10.1359	10.1731	10.2103	10.2476
55	10.4729	10.5100	10.5485	10.5864	10.6244
56	10.8537	10.8911	10.9306	10.9692	11.0079
57	11.2413	11.2804	11.3196	11.3582	11.3981
58	11.6357	11.6755	11.7153	11.7553	11.7953
59	12.0369	12.0773	12.1179	12.1585	12.1992
60	12.4449	12.4861	12.5273	12.5686	12.6100
61	12.8595	12.9016	12.9396	12.9856	13.0236
62	13.2773	13.3238	13.3623	13.4092	13.4478
63	13.7056	13.7528	13.7921	13.8396	13.8781
64	14.1407	14.1888	14.2285	14.2768	14.3166
65	14.5826	14.6316	14.6717	14.7208	14.7612
66	15.0412	15.0808	15.1218	15.1716	15.2126
67	15.4867	15.5372	15.5786	15.6292	15.6708
68	15.9490	16.0004	16.0422	16.0936	16.1358
69	16.4180	16.4700	16.5126	16.5648	16.6079
70	16.8939	16.9468	16.9899	17.0428	17.0861
71	17.3765	17.4303	17.4739	17.5280	17.5715
72	17.8660	17.9204	17.9647	18.0196	18.0637
73	18.3622	18.4176	18.4623	18.5180	18.5627
74	18.8653	18.9216	18.9667	19.0232	19.0684
75	19.3751	19.4340	19.4778	19.5352	19.5810
76	19.8918	19.9496	19.9959	20.0540	20.1024
77	20.4152	20.4740	20.5267	20.5796	20.6265
78	20.9459	21.0022	21.0524	21.1120	21.1595
79	21.4825	21.5428	21.5908	21.6512	21.6992
80	22.0264	22.0876	22.1360	22.1972	22.2458
81	22.5770	22.6392	22.6880	22.7504	22.7992
82	23.1344	23.1971	23.2467	23.3100	23.3593
83	23.6987	23.7624	23.8123	23.8764	23.9263
84	24.2697	24.3344	24.3847	24.4496	24.5000
85	24.8475	24.9132	24.9639	25.0296	25.0806
86	25.4322	25.4984	25.5499	25.6164	25.6679
87	26.0236	26.0908	26.1427	26.2100	26.2621
88	26.6218	26.6900	26.7423	26.8104	26.8620
89	27.2268	27.2956	27.3487	27.4176	27.4706
90	27.8387	27.9084	27.9618	28.0316	28.0851

An Explanation of the Table of Wine Areas.

The Explanation of the Table of Ale Areas may serve to explain this also, because the Diameters and Areas are placed alike in both Tables; for if you find any Diameter from 1 to 90 Inches in the first or seventh Column, and the Tenths (if any) at the Head of the Table, at the Angle of meeting you will have the Area required to the ten thousandth Part of a Wine Gallon.

And in case the given Diameter should exceed the Limits of the Table, then according to the Directions in the like Case for Ale Areas find the Area of half the given Diameter, which multiply by 4 and the Product will be equal to the Area required.

ART. V.

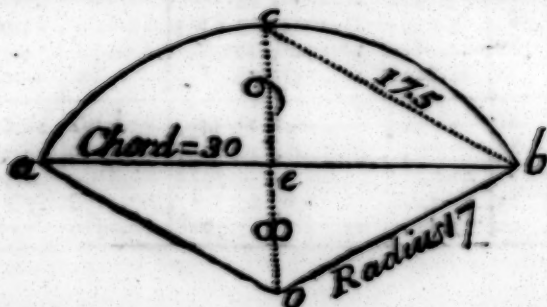
To find the Area of a Sector of a Circle.
See the following Figure.

DEFINITION.

A Sector of a Circle is a Figure contained between the two Semidiameters and Part of the Arch of a Circle, and is equal in Area to a Triangle whose Perpendicular is equal to the Radius, and the Base equal to the Arch Line of the Sector.

RULE.

Multiply the Radius or Semidiameter by half the Length of the Arch, or the whole Arch by the Radius, and the Product will be the Area in Inches; which divided by 282. 231. 2150.42, will give the Area in Ale and Wine Gallons, or Malt Bushels.



The Length of the Arch Line not being easily obtained by Measure, I shall give a practical Method for finding the same, by having the Length of the Chord of half the Arch given; which will come pretty near the Truth.

APPENDIX. 199

Of finding the Area of the Segment of a Circle.

To find the Length of the Arch multiply the Chord of half the Arch by 8; from that Product subtract the Chord of the whole Arch, and the Remainder divide by 3, and the Quotient will be nearly equal to the Length of the Arch.

EXAMPLE.

Let the Radius $bo = ao = co = 17$ Inches, the Chord of the whole Arch $ab = 30$ Inches, and the Chord of half the Arch $bc = 17.5$ Inches: To find the Area of the Sector $acbo$ in Square Inches.

OPERATION.

1. For the Length of the Arch acb .

Chord of $\frac{1}{2}$ the Arch $cb = 17.5$
Common Multiplier $= 8$

140.0

Chd. of whole Arch sub. 30

Remainder divide by 3 $110.0 (36.666 = \text{Arch } cb.$

2. For the Area of the Sector.

Length of the Area $acb = 36.666$
Multiply by $\frac{1}{2}$ the Radius $= 8.5$

183330

293328

Product $———— = 311,6610 = \text{Area of Sect.}$
in sq. Inches.

ART. VI.

To find the Area of the Segment of a Circle.

See the last Figure.

DEFINITION.

THE Segment of a Circle is the Part contained between the Chord ab and the Arch acb . Now it is plain from the Figure that if the Area of the Triangle abo be taken from the Area of the Sector found in the last Article the Remainder will be equal to the Area of the Segment $acbe$.

RULE.

Find the Area of the Sector by the Directions given in the last Article; then find the Area of the Triangle abo , which subtract from the former, and the Remainder will be the Area of the Segment required.

EXAMPLE.

Let the Dimensions be the same as in the last Article, *viz.* Radius $ab = ac = 17$ Inches, Chord $ab = 30$ Inches, Chord $cb = 17.5$ Inches, and the versed Sine ec , or Height of the Segment $= 9$ Inches, and the Perpendicular Height of the Triangle $oo = 8$ Inches: To find the Area of the Segment $acbe$ in Inches.

OPERATION.

Base of the Triangle $ab = 30$ $\frac{1}{2}$ Perpendicular $eo = 4$ Subtract the Product $= 120 =$ Area of Triang.
From Area of Sector $= 311.661$ $ab e.$ Remainder $= 191.661 =$ Area of Seg.
sq. Inches.

ART. VII.

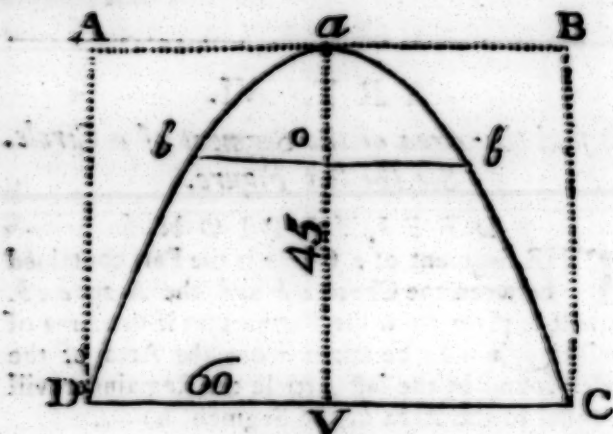
To find the Area of a Parabola. See the following Figure.

DEFINITION.

A Parabola is one of the Conic Sections, and is formed by cutting a Cone by a Plane passing parallel to the opposite Side of its Entrance, and is equal to $\frac{2}{3}$ of the Parallelogram that circumscribes it, viz. $Da c$ is equal to $\frac{2}{3}$ of $ABCD$.

RULE.

Multiply the Base or Ordinate dc by the Perpendicular or Axis av , and that Product divide by 1 and $\frac{1}{2}$ of the Square Divisors in Page 49; the Quotient will give the Area of the same Kind as your Divisor.



EXAMPLE.

Let the Ordinate or Base $dc = 60$ Inches, and the Axis or Perpendicular $av = 45$ Inches, what will the Area be in Ale Gallons?

OPERATION.

Base $dc = 60$ Perpendicular $av = 45$

$$\begin{array}{r} 300 \\ 240 \\ \hline \end{array}$$

$1 \frac{1}{2}$ of $282 = 423$ $2700(6.38 =$ Area in Ale Gs.
ART.

ART. VIII.

To find the Content of a Parabolic Conoid.

DEFINITION.

A Parabolic Conoid is generated by the Rotation of the Semiparabola dew about its Axe av , remaining fixed till the Motion end where it first began; and is equal to half its circumscribing Cylinder.

RULE.

Multiply the Square of the Diameter of the Base by the Height, and that Product divided by double of the Circular Divisors in Page 49 will give the Content in the same Denomination as the Divisor.

EXAMPLE.

Let the Diameter of the Base $dc = 60$ Inches, (see the last Figure) and the Height $av = 45$ Inches: To find the Content in Ale Gallons.

OPERATION.

$$\begin{array}{r}
 \text{Base } dc \text{ ---} = 60 \\
 \text{---} \\
 60 \\
 \text{---} \\
 \text{Square of } dc \text{ ---} = 3600 \\
 \text{Perpendicular } av \text{ ---} = 45 \\
 \text{---} \\
 18000 \\
 14400 \\
 \text{---}
 \end{array}$$

Content.

$$2 \times 359.05 = 718.1162000. (225.59 \text{ Ale Gs.})$$

ART. IX.

To find the Content of the Frustrum of a Parabolic Conoid.

DEFINITION.

THE Frustrum of a Parabolic Conoid is the lower Part of the Conoid cut off at bob parallel to the Base dvc . See the last Figure.

RULE.

Multiply the Sum of the Squares of the Diameters of the Ends by the Length, and that Product divided by Double of the Circular Divisors in Page 49 will give the Content in the same Kind as the Divisor.

Or find the Areas of the Diameters and add them together, and multiply their Sum by half the Height; the Product will be the Content.

EXAMPLE.

Admit the Frustrum's Height = vo to be = 30 Inches, and the least Ordinate or Diameter dod = 34.6 Inches, and the greatest dvc = 60 Inches as before, what will its Content be in Ale Gallons?

OPERATION.

$$\begin{array}{r}
 dod = 34.6 \quad dvc = 60 \\
 \underline{34.6} \qquad \qquad \underline{60} \\
 2076. \quad dvc \text{ sq.} = 3600 \\
 1384 \quad dod \text{ sq.} = 1197.16 \\
 1038 \quad \underline{\hspace{1cm}} \\
 \text{Sum} = 4797.16 \\
 1197.16 \quad vo = \underline{\hspace{1cm}} 30
 \end{array}$$

$$718,1)143914,80(200,41 = \text{Cont. in Ale Ga.}$$

$$\begin{array}{r}
 .29480 \\
 28724 \\
 \hline
 .7560 \\
 7181 \\
 \hline
 .379
 \end{array}$$

ART. X.

To find the Content of a Spheroid.

DEFINITION.

A Spheroid is generated by the Rotation of a Semiellipsis about its Transverse Diameter till the Motion end where it began, and is equal to $\frac{2}{3}$ of its circumscribing Cylinder.

RULE.

Multiply the Square of the Conjugate Diameter by the Transverse Diameter, and that Product divide by the Globular Divisors in Chap. viii. Art. vi. the Quotient will be the Content of the same Kind as the Divisor made use of.

EXAMPLE.

Let the Ellipsis in Chap. vii. Art. x. represent the Spheroid, whose Transverse Diameter is = 72 Inches, Conjugate Diameter 50 Inches: To find the Content in Ale Gallons.

OPERATION.

$$\begin{array}{r}
 \text{Conjugate Diam. } dc = 50 \\
 \underline{50} \\
 \text{Square of Conjugate Diam.} = 2500 \\
 \text{Transverse Diam. } ab = 72 \\
 \underline{72} \\
 5000 \\
 17500 \\
 \hline
 \text{Content.}
 \end{array}$$

$$\frac{2}{3} \text{ of } 359.05 = 538.58)180000.(334.21 \text{ A.G.}$$

In this Manner may the Content of the four last Figures be found in any other Denominations, provided you make use of the proper Divisors for that Purpose.

ART.

ART. XL

To find the Solidity of the Ungula or Hoofs of the Frustrum of a Cone, having the Length of the greatest and least Diameters, and also the Depth given in Inches.

Rule I. for the greater Hoof.

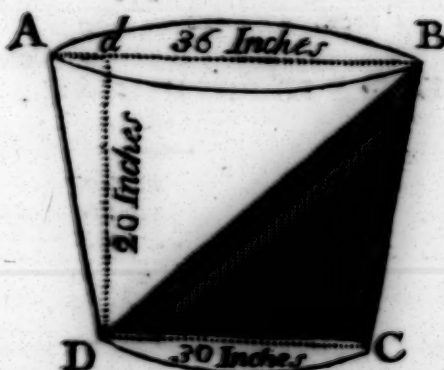
MULTIPLY the Product of the greater Diameter and the Frustrum's Height by the Square of the greater Diameter made less by the Product of the lesser Diameter, into a Mean Proportional between the two Diameters, and this last Product divide by three Times the Circular Divisors in Page 49 multiplied into the Difference of the Diameters, and the Quotient will be the Solidity of the greater Hoof.

Rule II. for the lesser Hoof.

Multiply the Product of the lesser Diameter and Height by the Product of the greater Diameter into a Mean Proportional between the two Diameters made less by the Square of the lesser Diameter, and this last Product divide by three Times the Circular Divisors in Page 49 multiplied into the Difference of Diameters, and the Quotient will be the Solidity of the lesser Hoof.

EXAMPLE.

Let the following Figure represent the Frustrum of a Cone, whose greater Diameter $AB = 36$ Inches, the less Diameter $DC = 30$ Inches, and the Height $dd = 20$ Inches; then the Mean Proportional will be nearly $= 32$ Inches, and the Difference of Diameters $= 6$ Inches: To find the Solidity of the Part $DA B$ the greater Hoof, and also that of DBC the lesser Hoof in Ale Gallons.



OPERATION;

APPENDIX.

OPERATION.

1. For the greater Hoof D A B.

$$\begin{array}{rcl} \text{Gr. Diam. AB} & = & 36 \\ \text{Height Dd} & = & 20 \end{array} \quad \begin{array}{rcl} 36 & \text{Mean Diam.} & = 32 \\ 36 & \text{Left Dm. DC} & = 30 \end{array}$$

$$\begin{array}{rcl} 1 \text{ Prod.} & \text{---} & = 720 \\ & & 216 \end{array} \quad \begin{array}{rcl} 2 \text{ Prod.} & \text{---} & = 960 \\ & & 108 \end{array}$$

$$\text{AB Square} = 1296$$

$$2 \text{ Prod. sub.} = 960$$

$$\text{Remains ---} = 336$$

$$1 \text{ Prod. ---} = 720$$

$$6720$$

$$2352$$

$$\text{Last Prod.} = 241920$$

PRODUCT.

$$3 \times 359 \times 6 = 6462)241920(37.43 = \text{Cont. in Ale Gs.}$$

$$48060$$

$$45234$$

$$28360$$

$$25848$$

$$24120$$

$$19386$$

$$\text{Remains ---} = 4734$$

2. For the lesser Hoof D B C.

$$\begin{array}{rcl} \text{Left D. DC} & = & 30 \\ \text{Height Dd} & = & 20 \end{array} \quad \begin{array}{rcl} \text{Gr. D. AB} & = & 36 \\ \text{Mean D.} & = & 32 \end{array} \quad \begin{array}{rcl} 30 \\ 30 \end{array}$$

$$\begin{array}{rcl} 1 \text{ Prod.} & = & 600 \\ & & 108 \end{array} \quad \begin{array}{rcl} \text{DC sq} & = & 900 \end{array}$$

$$2 \text{ Prod. ---} = 1152$$

$$\text{DC squared sub.} = 900$$

$$\text{Remains ---} = 252$$

$$1 \text{ Prod. ---} = 600$$

Content.

$$3 \times 359 \times 6 = 6462)151200(23.39 = \text{Ale Gs.}$$

$$21960$$

$$19386$$

$$25740$$

$$19386$$

$$63540$$

$$58158$$

$$\text{Remains ---} = 5382 \quad \text{Thus}$$

Thus I have found the Content of the greater Hoof $DAB = 37.43$ Ale Gallons, and that of the lesser Hoof $DBC = 23.39$ Ale Gallons, whose Difference is 14.04 Gallons; which exhibits the Error of taking half the Content of a Cone's Frustrum for the Solidity of the Drip of a Tun, which in Fact is either the greater or lesser Hoof of the Frustrum according to its Position, whether it stands upon its greater or lesser Base.

This Method is very correct, and may easily be put in Practice in finding the Drip of a conical Tun; for if you take a Diameter at the Base, and another parallel to that, where the Water cuts the Side in the deepest Place when the Bottom is just covered, and also the perpendicular Height, you will have a little Frustrum divided into two Hoofs, when if the Tun stands upon the lesser Base the Drip will be the lesser Hoof, but if upon the greater Base then the Drip will be the greater Hoof.

ART. XII.

To find the Content of the Ungula or Hoofs of the Frustrum of a square Pyramid, having the Length of the greatest and least Sides, and the Depth given in Inches.

Rule I. for the greater Hoof.

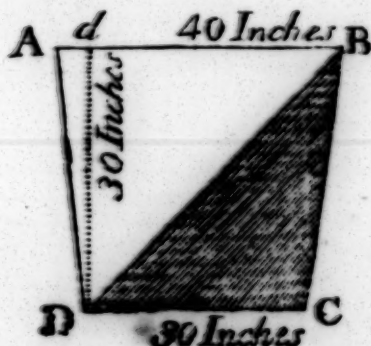
1. **T**O twice the Square of the greater Side add the Product of the greater and lesser Sides, their Sum multiply into the Depth, and this Product divide by 6 Times the Square Divisors in Page 49; the Quotient will be the Solidity of the greater Hoof.

Rule II. for the lesser Hoof.

2. To twice the Square of the lesser Side add the Product of the greatest and least Sides, their Sum multiply into the Depth, and this Product divide by 6 Times the Square Divisors in Page 49; the Quotient will be the Solidity of the lesser Hoof.

EXAMPLE.

Let the following Figure $ABCD$ represent the Frustrum of a square Pyramid, whose greater Side $AB = 40$ Inches, lesser Side $DC = 30$ Inches, and the Depth $AD = 20$ Inches: To find the Content of the Hoofs DAB and DBC in Ale Gallons.



1. For the greater Hoof D A B.

OPERATION.

$$\begin{array}{rcl}
 AB & \text{---} & = 40 \quad AB = 40 \\
 & & 40 DC = 30 \\
 \hline
 \text{Square of } AB & = 1600 & \text{Prod.} = 1200 \\
 & & 2 \text{ Add} = 3200 \\
 \hline
 \text{Twice Sq. of } AB & = 3200 & \text{Sum} = 4400 \\
 \text{Multiply by } dD & \text{---} & = 20 \\
 \hline
 6 \times 282 & \text{---} & = 1692 \quad \text{Content.} \\
 & & 88000(52.01 = \\
 & & 8460 \quad DAB. \\
 \hline
 & & .3400 \\
 & & 3384 \\
 \hline
 & & .1600 \\
 & & 1692 \\
 \hline
 \text{Remains} & \text{---} & 92
 \end{array}$$

2. For the lesser Hoof D B C.

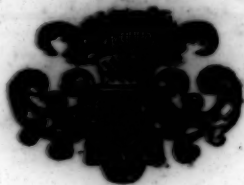
OPERATION.

$$\begin{array}{rcl}
 DC & \text{---} & = 30 \quad AB = 40 \\
 & & 30 DC = 30 \\
 \hline
 \text{Square of } DC & = 900 & \text{Prod.} = 1200 \\
 & & 2 \text{ Add} = 1800 \\
 \hline
 \text{Twice Sq. of } DC & = 1800 & 3000 \\
 \text{Multiply by } dD & \text{---} & = 20 \\
 \hline
 6 \times 282 & = 1692 & \text{Content.} \\
 & & 60000(35.46 = \\
 & & 5076 \quad DBC. \\
 \hline
 & & .9240 \\
 & & 8460 \\
 \hline
 & & .7800 \\
 & & 6768 \\
 \hline
 & & 10320 \\
 & & 10152 \\
 \hline
 \text{Remains} & \text{---} & .168
 \end{array}$$

The Content of the greater Hoof $DAB = 52.01$ added to the Content of the lesser Hoof $DAC = 35.46$ is equal to 87.47 Ale Gallons equal to the whole Content of the Frustrum $ABCD$.

These two Rules will be of good Use in finding the Drip of any Square Pyramidal Tun; for if the Tun stands slanting upon the greater Base, then the Drip will form the greater Hoof; but if upon the lesser Base, the Drip will form the lesser Hoof of a Frustrum, cut off where the Water touches the Side, in the deepest Place when the Bottom is just covered. Therefore if the Dimensions of the Sides be taken between every 6 or 10 Inches of the Depth, agreeable to the Directions given in Chap. x. Art. vi. for a Conical Tun, and the Drip be found according to the Rules given in this Article, then any Tun of this Form may be easily inched, and tabulated so as to give the Content at any Depth.

F I N I S.



AN
ESSAY

Towards a PRACTICAL

English Grammar,

DESCRIBING THE

GENIUS and NATURE

OF THE

ENGLISH TONGUE;

Giving likewise

A Rational and Plain ACCOUNT of
GRAMMAR in General, with a fa-
miliar EXPLANATION of its Terms.

By JAMES GREENWOOD,
Sur-Master of St. Paul's-School.

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